# Supplemental Material: Multi-Scale Modeling and Rendering of Granular Materials


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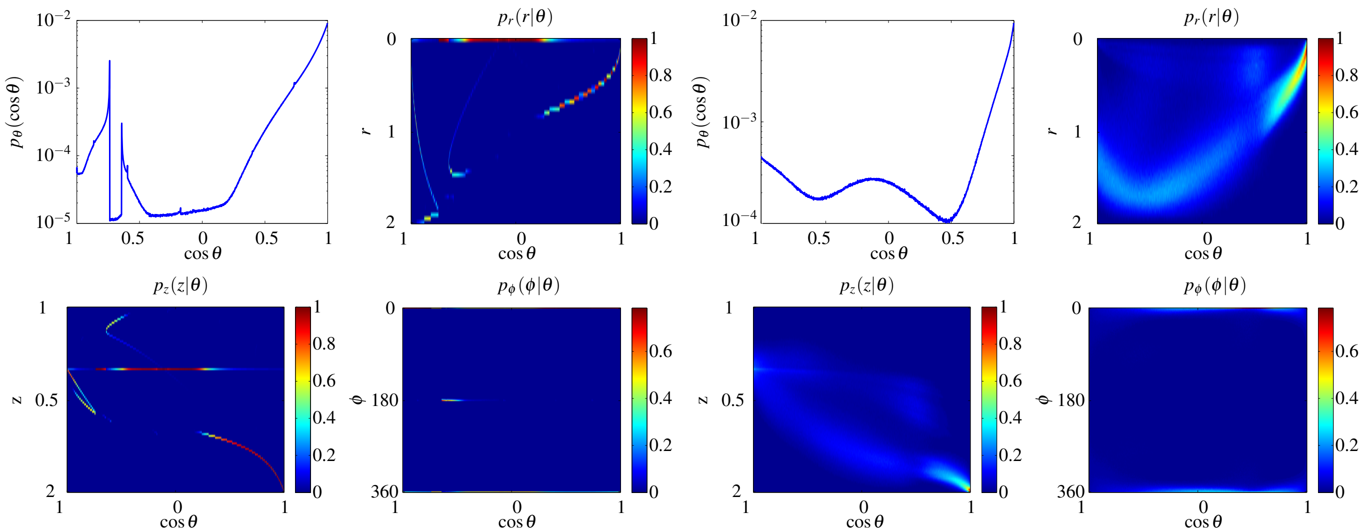


**Figure 1:** *The TSDF of a dielectric sphere with index of refraction $\eta = 1.33$ (left two columns) and our snow grain (right two columns). The arc-like structures in the plots are due to internal reflections. Note that the graph for $p_\theta(\cos\theta)$ for the dielectric sphere has been validated against literature (cp. [Zhou et al. 2003, Fig. 8]).*

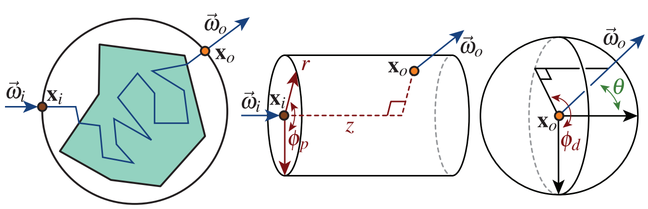


**Figure 2:** *Our notation as well as the cylindrical and spherical coordinate systems used for TPT.*

## 1 Teleportation Path Tracing

In this section, we will describe *Teleportation Path Tracing* (TPT), an implementation of Teleportation Transport (TT) as described in the main paper. We will also detail some of the implementation challenges that arise due to the non-point scattering nature of granular media.

**Sampling free paths** TPT uses the same free path model as was described for VPT in the main paper. In particular this means that the delta forward scattering component of grains due to the hit probability $\beta$ need not be handeled during scattering, but is accounted for by free path sampling.

**Coordinate systems** Like Volumetric Path Tracing (VPT), TPT combines two sampling processes: First, a free path to the next path vertex is sampled from an exponential distribution. Second, a directional distribution is sampled to obtain the direction after scattering. In contrast to VPT, however, this step also includes non-point scattering, which we refer to as "teleportation". Thus, in addition to an outgoing direction $\vec{\omega}_o$, a outgoing position $\vec{x}_o$ must be sampled.

We express each outgoing position $\mathbf{x}_o$ in a cylindrical coordinate system (see Figure 2) centered around the incoming ray resulting in the three coordinates $\mathbf{x}_o = (r, \phi_p, z)$. We express the outgoing direction in spherical coordinates as $\vec{\omega}_o = (\theta, \phi_d)$, also aligned with the incoming direction, so $\cos(\theta) = \vec{\omega}_i \cdot \vec{\omega}_o$. However, because the orientation of the grain is random, the orientation of these coordinate systems around the incoming ray is arbitrary, so only the angle $\phi_\Delta = \phi_p - \phi_d$ is relevant.

**TSDF** The 4D distribution $p(\theta, \phi_\Delta, r, z)$ is the *teleportation scattering distribution function* (TSDF). In other words, this is the distribution of random offsets and angles that summarizes the interactions with grains, relative to their bounding spheres, when the positions and orientations of the grains are ignored.

Tabulating the full 4D distribution is an unnecessarily large amount of data to store. We have found that for the grains we cosidered the $\phi_\Delta$, $r$, and $z$ dimensions tend to be uncorrelated. Therefore, we reduce the whole 4D distribution to a tabulated 1D distribution $p_\theta(\theta)$ and three 2D conditional distributions $p_{\phi_\Delta}(\phi_\Delta|\theta)$, $p_r(r|\theta)$, and $p_z(z|\theta)$:

$$p(\theta, \phi_\Delta, r, z) \approx p_\theta(\theta) p_{\phi_\Delta}(\phi_\Delta|\theta) p_r(r|\theta) p_z(z|\theta). \qquad (1)$$

During directional sampling, we first sample $\theta$ and then sample

*The work was done while the author was employed at Disney Research.

**Algorithm 1** Sampling $\vec{\omega}_o$ and $\mathbf{x}_o$ in TPT

**function** SAMPLETSDF($\vec{\omega}_i$)
  $\phi_d = \text{SampleUniform}(0, 2\pi)$
  $\theta = \text{Sample}(p_\theta)$
  $r = \text{Sample}(p_r)$
  $z = \text{Sample}(p_z)$
  $\phi_\Delta = \text{Sample}(p_{\phi_\Delta})$
  $\vec{\omega}_o = R_{\vec{\omega}_i}(\phi_d)\left(\cos(\theta)\vec{\omega}_i + \sin(\theta)\vec{\omega}_i^{\perp}\right)$
  $\mathbf{x}_o = R_{\vec{\omega}_i}(\phi_d + \phi_\Delta)\left(z\vec{\omega}_i + r\vec{\omega}_i^{\perp}\right)$
**end function**

$\phi_\Delta, r, z$ independently. This process is shown in Algorithm 1. Examples of TSDF for dielectric spheres and snow grains are shown in Figure 1.

**Implementation Challenges** The non-point nature of teleportation poses practical problems. For example, a ray that is inside the granular medium may be teleported outside, or into geometry that intersects the granular medium. This leads to light leaking, and complicates tracking the current medium.

We devised the following solution for this problem. First, we find collisions using a ray intersection test from $\mathbf{x}_i$ to $\mathbf{x}_o$. If this ray intersects any geometry other than the bounding mesh of the granular medium, we reset the ray to the intersection point and continue using normal surface vertex handling (e.g. reflection in the case of a diffuse surface). Similarly, if the ray exits the bounding mesh, we reset to the boundary and handle it as an indexed-matched boundary.

## References


ZHOU, X., LI, S., AND STAMNES, K. 2003. Geometrical-optics code for computing the optical properties of large dielectric spheres. *Applied optics 42*, 21, 4295–4306.