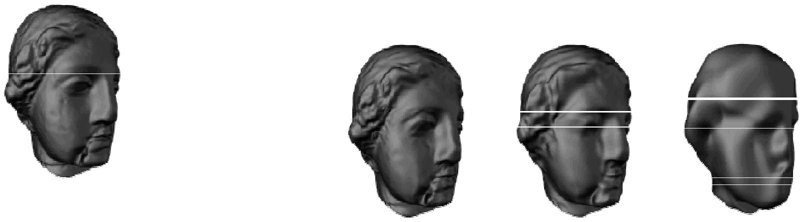
Progressive Geometry Compression

Andrei Khodakovsky Peter Schröder Wim Sweldens



Motivation

- Large (up to billions of vertices), finely detailed, arbitrary topology surfaces
- Difficult manageability of such large amounts of information
  - Computation, Storage, Transmission, Display Strain

Progressive Compression is needed

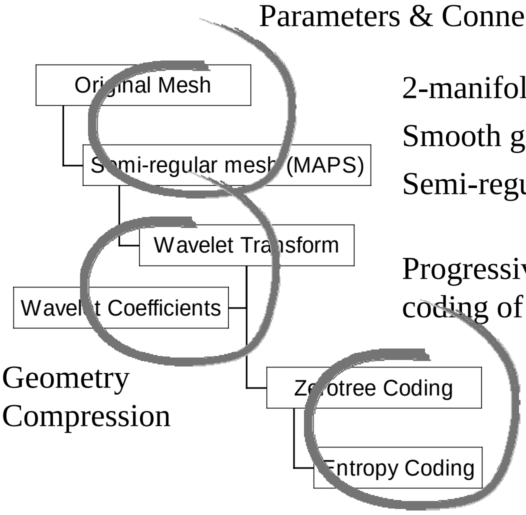
What Compression Is All About

- Accuracy/ bit per vertex
- Definition of a Geometry Error
  - Measure of the geometric distance between 3D objects
- New Problems with Respect to Image Compression
  - No direct correspondence between original and compressed surface

Algorithm

Detailed Mesh

Parameters & Connectivity



2-manifold, arbitrary connectivity

Smooth global parameterization

Semi-regular approximation

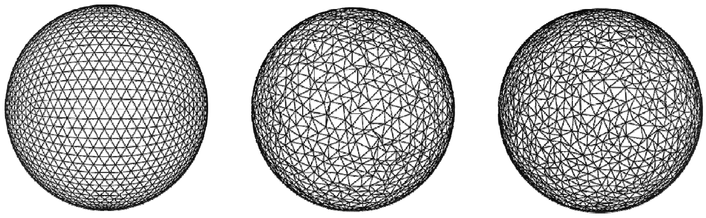
Progressive transformation and coding of the approximated shapes

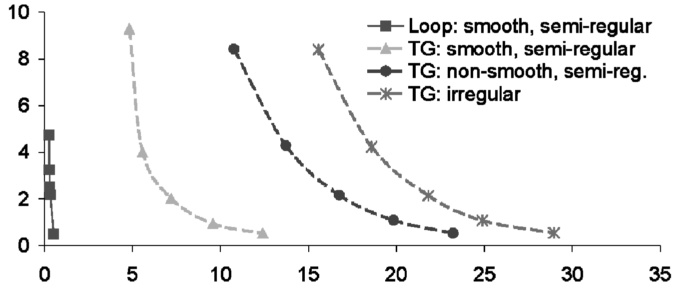
Geometry Compression

Bitrate

Coarse Mesh

Geometry-Parameter-Connectivity



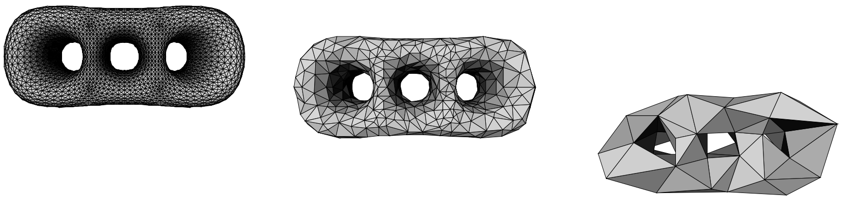


Geometry Error

- For highly detailed, densely sampled meshes, the sample location (vertices + connectivity) do not contribute in improving the geometric distance, and thus the error
- Parameter information is thus contained within the surface, and any rate-distortion improving coding should better take advantage from the geometric information only, normal to the surface

MAPS algorithm

- "Multiresolution Adaptive Parametrization of Surfaces" based on edge collapsing



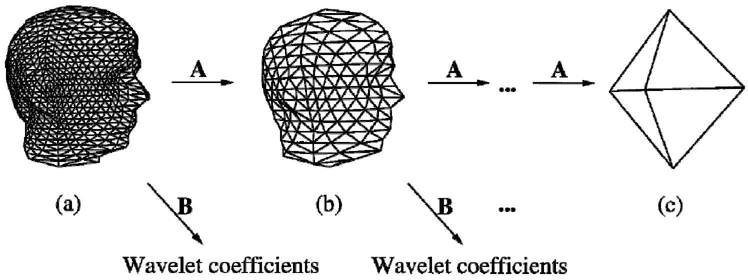
Semi-regular approximation is achieved on applying triangle quadrisection to the coarse mesh, and taking advantage of the mapping operated by the algorithm between original and coarse mesh.

Wavelet Transform-1

modeling of a complex object of arbitrary topology at multiple levels of detail

- Replacement of level-N mesh with coarser level-(N-1) mesh + wavelet coefficients
- Generation of "nested" meshes
- Subdivision Rules and Filter-Banks

Wavelet Transform-2



Semi-regular mesh is hierarchically subdivided
into a coarser mesh and some details information

Reconstruction is achieved from the coarse mesh by
hierarchical addition of detail information

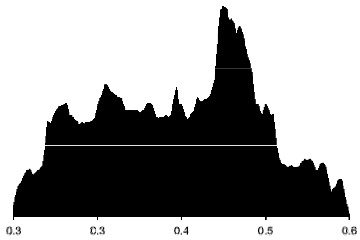
Wavelet Transform-3

- Filter Bank Algorithm
  - Design of Analysis and Synthesis Filters.
    - Decomposition of a mesh like $S^{j+1}(\mathbf{x}) = \sum_i v_i^j \phi_i^j(\mathbf{x}) + \sum_i w_i^j \psi_i^j(\mathbf{x})$
- Our Paper:
  - Synthesis: $$\mathbf{p}^{j+1} = \begin{bmatrix} \mathbf{P} & \mathbf{Q} \end{bmatrix} \begin{bmatrix} \mathbf{p}^j \\ \mathbf{d}^j \end{bmatrix}$$
  - Analysis: solution [pj,dj] of system for a given pj+1

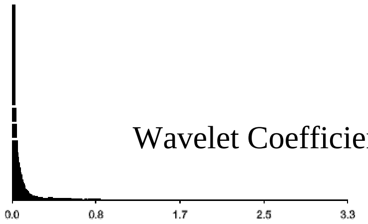
  - P, low-pass reconstruction filter
  - Q, high-pass reconstruction filter
  - small support

Wavelet Transform-4

Decorrelating effect of the Wv Transform



Vertex position magnitudes for “Venus”



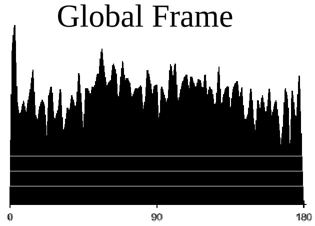
Wavelet Coefficients Magnitudes

Geometry Compression

Wavelet Coefficients

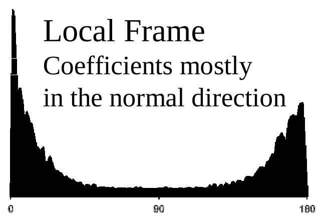
- Vector valued
- x,y,z components pretty correlated, but
- decorrelation in a local frame

Global Frame



Local Frame

Coefficients mostly in the normal direction



- Normal component relevant for geometric information
- Code each component independently

Hierarchical Trees

- Exploit the relationship of wavelet coefficients across bands, and their exponential decay
- Areas with significant information are similar in shape and location
- Non significance in a low frequency band for a particular level of accuracy means with high probability non significance of the children nodes.
- Localizing a zerotree avoids transmitting a large amount of insignificant details with respect to a desired level of accuracy
- Progressive compression, embedded code

Zerotree Coding

Example of the quality of the coding

SPIHT PSNR = 35.12 dB, JPEG PSNR = 31.8 dB (quality factor 15%).

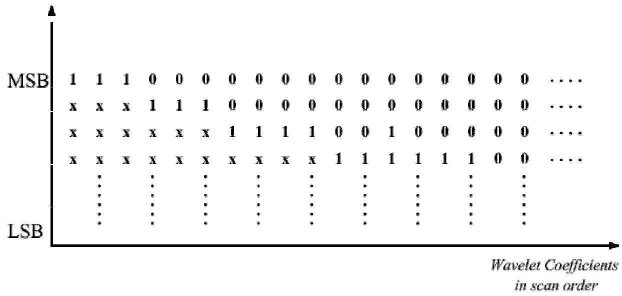
SAID AND PEARLMAN, A New, Fast, and Efficient Image Codec Based on Set Partitioning in Hierarchical Trees. *IEEE Transaction on Circuits and Systems for Video Technology* 6, 3 (1996), 243–250.

Zerotree-Howto

- General: let the decoder see just as much coefficients as needed

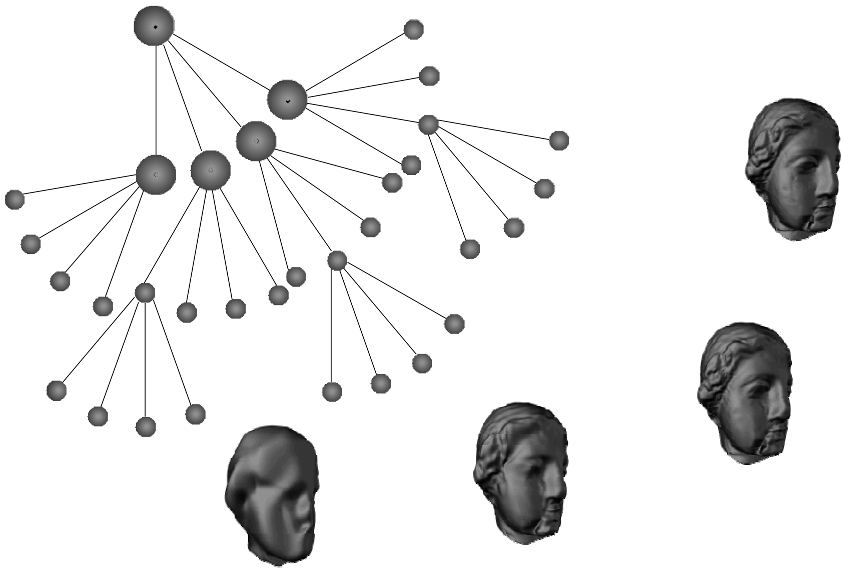
- Significance Map: decide which coefficient become significant at a particular bitrate
  – exponentially decreasing threshold
- Refinement Map: decide which coefficients need to be transmitted, as they became significant in a previous pass
- Sign Map: additional information for sign coding

Zerotree-Structure



Moves:
- wavelet coefficients are coded as 1-dim array
- lower frequency bands before higher frequency bands
- scan the array and select against a threshold value, which is repeatedly refined
- transmit zerotree information, i.e. as soon as a zero is encountered the corresponding coefficient node is treated as non-relevant.
- as long as a coefficient remains irrelevant a zerotree symbol will be transmitted for it

Zerotree-Example

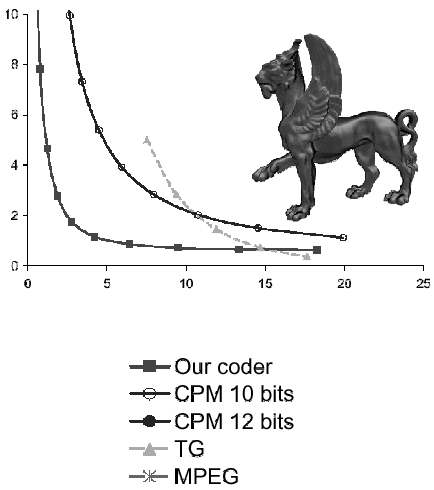


Entropy Coding

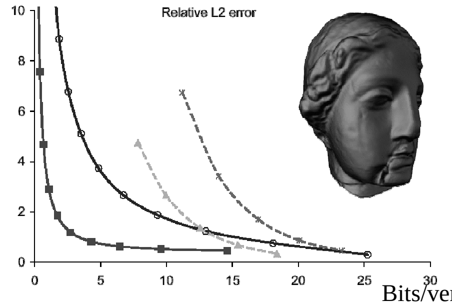
- Further improvement of the bit budget by eliminating redundancy due to non-uniform distribution
- Refinement and sign bits are found to be distributed uniformly
- Significance is a function of bitplane: “early” bitplanes will contain many insignificant coefficients, which will become significant in “later” bitplanes

Results

Rate Distortion Curves:
Improvement by a factor 4!

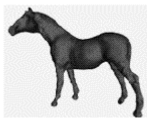


Relative L^2 error (10^{-4})

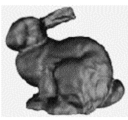


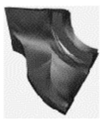
Bits/vertex

Live Compression...



476 B (e 40)
1528 B (e 12)
4163 B (e 4.7)
26800 B (e 0.82)





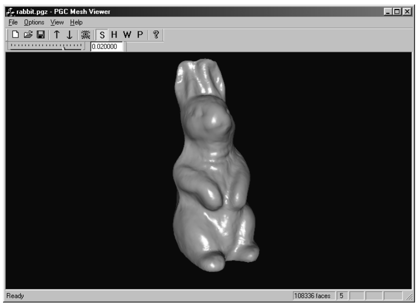
Conclusions

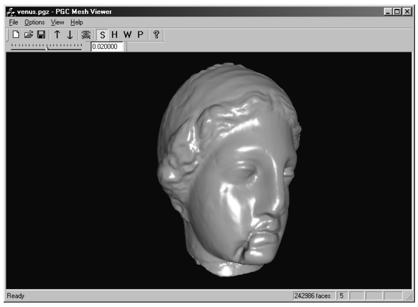
- Very effective compression algorithm
  - smooth appearance, low (hardware) strain
- Many details in very early stages of decompression
- Very fruitful distinction between parameters and geometry
- Still at the beginning of wavelet 3D model compression...

In Fact...

- Effectiveness of compression inherits very much from a bunch of “tested” approaches
- No sound treatment of multiresolution analysis
  - orthogonality, stability...
- “Tentative” choice of wavelets
- Elimination of tangential information (Normal Meshes...)

Demo





Thanks

- Andreas Hubeli for the supervision and for help
- You for your attention….

It’s your turn...

Extra Slides...

Wavelet Transform-2

Ingredients of Multiresolution Analysis (Mallat and Meyer) : Existence of nested linear spaces and of an inner product relative to a subdivision rule.

$$V^0 \subset V^1 \subset V^2 \subset \cdots$$

$$W^j := \{f \in V^{j+1} | \langle f, g \rangle = 0 \quad g \in V^j\} \qquad f^{j+1} = f^j + h^j$$

Nested spaces are generated by translations and dilations of a single function, $\phi(x)$

$$\phi(x) = \sum_i p_i \phi(2x - i)$$

$$V^j := \mathbf{Span}\{\phi(2^j x - i) | i = -\infty, \ldots, \infty\}$$

Subdivision Rules can be used to define such functions

Distance Function

- Euclidean Distance (L^2) d(X,Y) between two surfaces X, Y

$$d(X,Y) = \left(\frac{1}{\text{area}(X)} \int_{x \in X} d(x,Y)^2 dx \right)^{1/2}$$

- Symmetrize by taking the max of d(X,Y) and d(Y,X)