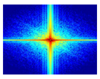
Spectral Processing of Point-Sampled Geometry

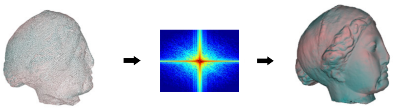


Overview

- Introduction
- Fourier transform
- Spectral processing pipeline
- Applications
  - Spectral filtering
  - Adaptive subsampling
- Summary

Introduction

- Idea: Extend the Fourier transform to manifold geometry



⇨ Spectral representation of point-based objects

⇨ Powerful methods for digital geometry processing

Introduction

- Applications:
  - Spectral filtering:
    - Noise removal
    - Microstructure analysis
    - Enhancement
  - Adaptive resampling:
    - Complexity reduction
    - Continuous LOD

Fourier Transform

- 1D example:

$$X_n = \sum_{k=1}^{N} x_k e^{-j2\pi\frac{nk}{N}}$$

spectral basis function

output signal

input signal

- Benefits:
  - Sound concept of frequency
  - Extensive theory
  - Fast algorithms

Fourier Transform

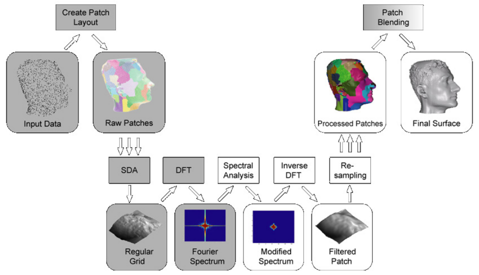
- Requirements:
  - Fourier transform defined on Euclidean domain
    ⇨we need a global parameterization
  - Basis functions are eigenfunctions of Laplacian operator
    ⇨requires regular sampling pattern so that basis functions can be expressed in analytical form (fast evaluation)
- Limitations:
  - Basis functions are globally defined
    ⇨Lack of local control

Approach

- Split model into patches that:
  - are parameterized over the unit-square
    - ⇨ mapping must be continuous and should minimize distortion
  - are re-sampled onto a regular grid
    - ⇨ adjust sampling rate to minimize information loss
  - provide sufficient granularity for intended application (local analysis)

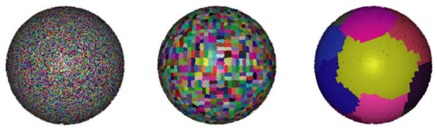
⇨ process each patch individually and blend processed patches

Spectral Pipeline



Patch Layout Creation

Clustering ⇨ Optimization



Samples ⇨ Clusters ⇨ Patches

Patch Layout Creation

- Iterative, local optimization method
- Merge patches according to quality metric:

$$\Phi = \Phi_S \cdot \Phi_{NC} \cdot \Phi_B \cdot \Phi_{Reg}$$

Φ_S ⇨ patch Size

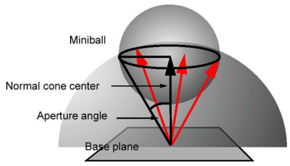
Φ_{NC} ⇨ curvature

Φ_B ⇨ patch boundary

Φ_{Reg} ⇨ spring energy regularization

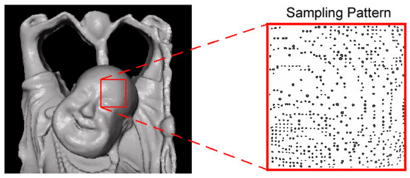
Patch Layout Creation

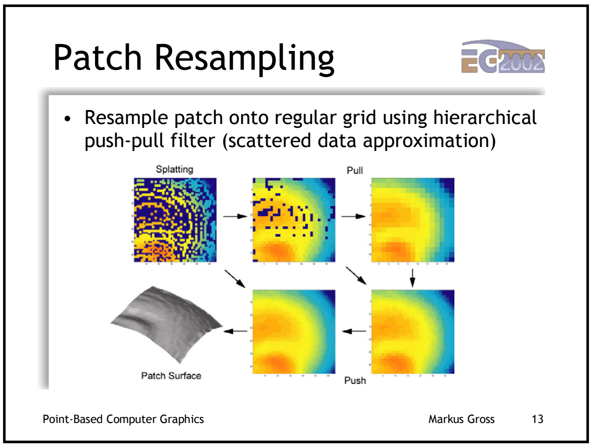
- Parameterize patches by orthogonal projection onto base plane
- Bound normal cone to control distortion of mapping using smallest enclosing sphere



Patch Resampling

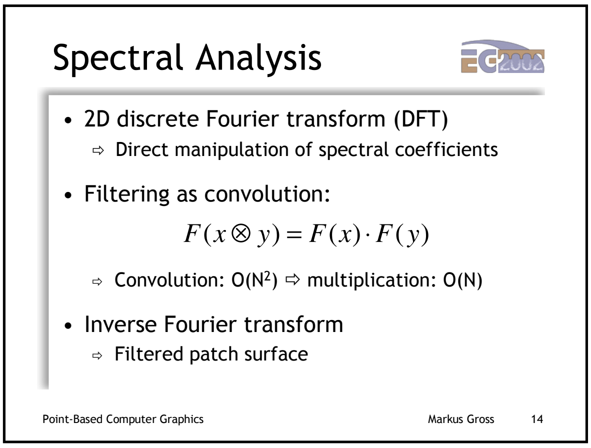
- Patches are irregularly sampled:





Patch Resampling

- Resample patch onto regular grid using hierarchical push-pull filter (scattered data approximation)

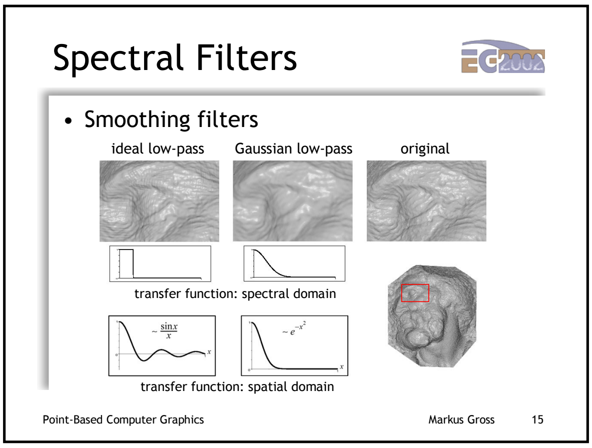


Spectral Analysis

- 2D discrete Fourier transform (DFT)
  - ⇨ Direct manipulation of spectral coefficients
- Filtering as convolution:

$$F(x \otimes y) = F(x) \cdot F(y)$$

  - ⇨ Convolution: O(N^2) ⇨ multiplication: O(N)
- Inverse Fourier transform
  - ⇨ Filtered patch surface

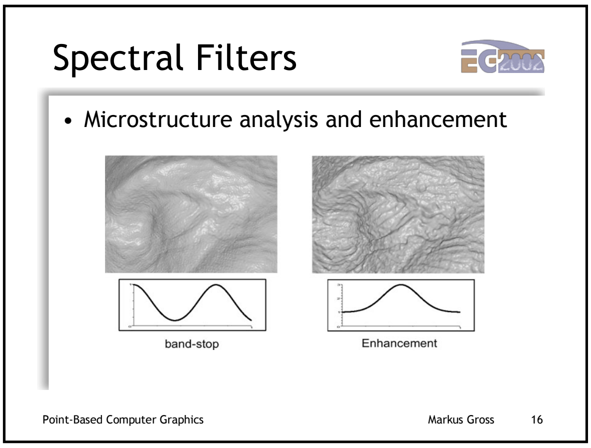


Spectral Filters

- Smoothing filters

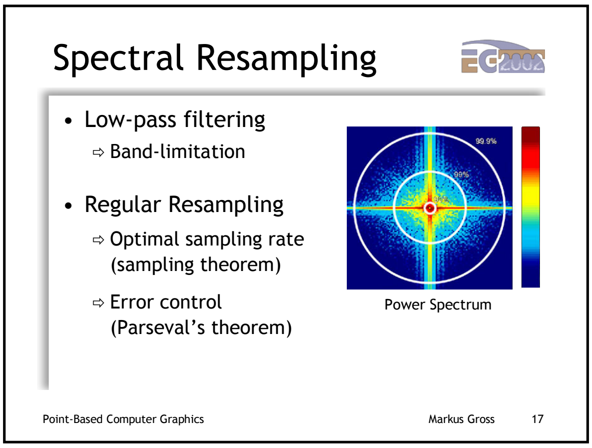
transfer function: spectral domain

transfer function: spatial domain



Spectral Filters

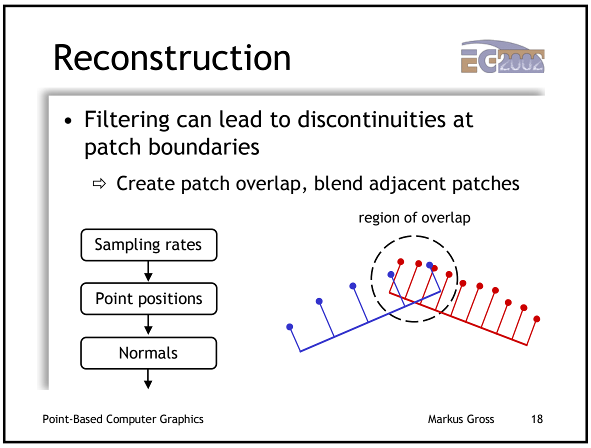
- Microstructure analysis and enhancement



Spectral Resampling

- Low-pass filtering
  - ⇨ Band-limitation
- Regular Resampling
  - ⇨ Optimal sampling rate (sampling theorem)
  - ⇨ Error control (Parseval's theorem)

Power Spectrum

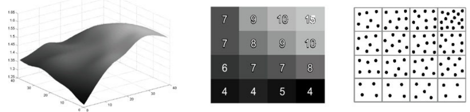


Reconstruction

- Filtering can lead to discontinuities at patch boundaries
  - ⇨ Create patch overlap, blend adjacent patches

Reconstruction

- Blending the sampling rate



blended sampling rate in region of patch overlap → discretized sampling rate on regular grid → pre-computed sampling patterns

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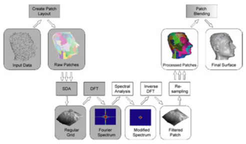
Timings

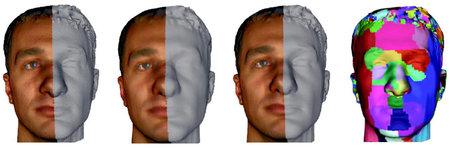


Time ↓

Clustering	9%
Patch Merging	38%
SDA	23%
Analysis	4%
Reconstruction	26%

Applications

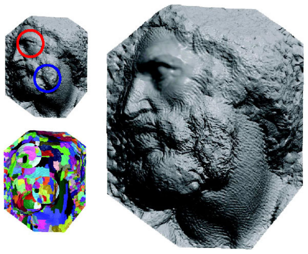
- Surface Restoration



Original Gaussian low-pass Wiener filter Patch layout

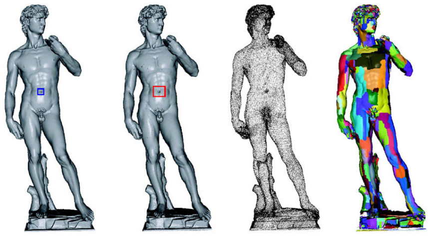
Applications

- Interactive filtering



Applications

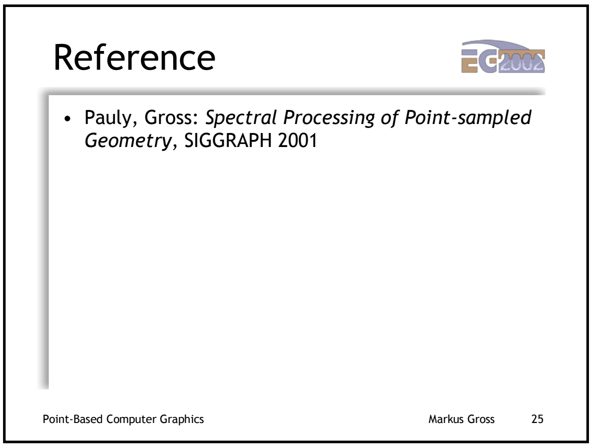
- Adaptive Subsampling



4,128,614 pts. = 100% 287,163 pts. = 6.9%

Summary

- Versatile spectral decomposition of point-based models
- Effective filtering
- Adaptive resampling
- Efficient processing of large point-sampled models



Reference

- Pauly, Gross: *Spectral Processing of Point-sampled Geometry*, SIGGRAPH 2001