Computer Graphics
Direct Illumination II

Dr. Marios Papas
marios@disneyresearch.com
Recap: Direct Illumination

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_e(r(x, \vec{\omega}_i), -\vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]
Recap: Direct Illumination

\[ L_r(x, z) = \int_A f_r(x, y, z) L_e(y, x)V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} dA(y) \]
Recap: Direct Illumination

\[ L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{\left| \cos \theta_i \right| \left| \cos \theta_o \right|}{\| x - y \|^2} dA(y) \]
Recap: Direct Illumination

\[ L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} dA(y) \]
Recap: Direct Illumination

\[ L_r(x, z) = \int_{A_e} f_r(x, y, z)L_e(y, x)V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} \, dA(y) \]
Recap: Direct Illumination

Sampling the hemisphere
Recap: Direct Illumination

Sampling the area of the light
Today’s Menu

• Importance Sampling
• Multiple Importance Sampling
• Environment lighting (image-based lighting)
• Marginal/conditional inversion sampling
• Hierarchical sample warping
• Product sampling
• Depth of field, motion blur
Importance Sampling
(re cap)
Importance Sampling

• Placing samples intelligently reduces variance

$$\left< L_r(x, \vec{\omega}_r)^N \right> = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(x, \vec{\omega}_{i,k}, \vec{\omega}_r) L_i(x, \vec{\omega}_{i,k}) \cos \theta_{i,k}}{p_\Omega(\vec{\omega}_{i,k})} d\vec{\omega}_{i,k}$$
Importance Sampling

• Placing samples intelligently reduces variance

\[
\langle L_r(x, \omega_r)^N \rangle = \frac{1}{N} \sum_{k=1}^{N} \frac{f_r(x, \omega_{i,k}, \omega_r) L_i(x, \omega_{i,k}) \cos \theta_{i,k}}{p_{\Omega}(\omega_{i,k})} \ d\omega_{i,k}
\]
Reflection Equation

• Which terms can we importance sample?

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

– BRDF
– incident radiance
– cosine term
Reflection Equation

• Which terms can we importance sample?

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i d\bar{\omega}_i \]

– BRDF
– incident radiance
– cosine term
Sampling the Cosine Term
Sampling the Cosine Term

• Let’s consider a simplified setup: diffuse objects illuminated by an ambient white sky
Sampling the Cosine Term

- Let’s consider a simplified setup: diffuse objects illuminated by an ambient white sky

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]
Sampling the Cosine Term

- Let’s consider a simplified setup: diffuse objects illuminated by an ambient white sky

\[
L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i
\]

\[
L_r(x) = \frac{\rho}{\pi} \int_{H^2} V(x, \omega_i) \cos \theta_i \, d\omega_i
\]
Sampling the Cosine Term

• Let’s consider a simplified setup: diffuse objects illuminated by an ambient white sky

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

\[ L_r(x) = \frac{\rho}{\pi} \int_{H^2} V(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

• a.k.a. *ambient occlusion*
Sampling the Cosine Term

• Let’s consider a simplified setup: diffuse objects illuminated by an ambient white sky

\[ L_r(x, \vec{\omega}_r) = \int_{H^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

\[ L_r(x) = \frac{\rho}{\pi} \int_{H^2} V(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]

• a.k.a. *ambient occlusion*
Ambient Occlusion
Ambient Occlusion

\[ L_r(x) = \frac{\rho}{\pi} \int_{H^2} V(x, \vec{\omega}_i) \cos \theta_i \, d\vec{\omega}_i \]
Ambient Occlusion

\[ L_r(x) = \frac{\rho}{\pi} \int_{H^2} V(x, \omega_i) \cos \theta_i \, d\omega_i \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \vec{\omega}_{i,k}) \cos \theta_{i,k}}{p(\vec{\omega}_{i,k})} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \omega_{i,k}) \cos \theta_{i,k}}{p(\omega_{i,k})} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k}}{p(\bar{\omega}_{i,k})} \]

Uniform hemispherical sampling
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \omega_{i,k}) \cos \theta_{i,k}}{p(\omega_{i,k})} \]

Uniform hemispherical sampling

\[ p(\omega_{i,k}) = \frac{1}{2\pi} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \vec{w}_{i,k}) \cos \theta_{i,k}}{p(\vec{w}_{i,k})} \]

Uniform hemispherical sampling

\[ p(\vec{w}_{i,k}) = \frac{1}{2\pi} \]

\[ L_r(x) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(x, \vec{w}_{i,k}) \cos \theta_{i,k} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k}}{p(\bar{\omega}_{i,k})} \]

Uniform hemispherical sampling

\[ p(\bar{\omega}_{i,k}) = 1/2\pi \]

Cosine-weighted importance sampling

\[ L_r(x) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k}}{p(\bar{\omega}_{i,k})} \]

**Uniform hemispherical sampling**

\[ p(\bar{\omega}_{i,k}) = \frac{1}{2\pi} \]

\[ L_r(x) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k} \]

**Cosine-weighted importance sampling**

\[ p(\bar{\omega}_{i,k}) = \frac{\cos \theta_{i,k}}{\pi} \]
Ambient Occlusion

\[ L_r(x) \approx \frac{\rho}{\pi N} \sum_{k=1}^{N} \frac{V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k}}{p(\bar{\omega}_{i,k})} \]

Uniform hemispherical sampling

\[ p(\bar{\omega}_{i,k}) = \frac{1}{2\pi} \]

\[ L_r(x) \approx \frac{2\rho}{N} \sum_{k=1}^{N} V(x, \bar{\omega}_{i,k}) \cos \theta_{i,k} \]

Cosine-weighted importance sampling

\[ p(\bar{\omega}_{i,k}) = \frac{\cos \theta_{i,k}}{\pi} \]

\[ L_r(x) \approx \frac{\rho}{N} \sum_{k=1}^{N} V(x, \bar{\omega}_{i,k}) \]
Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

1 sample/pixel
Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

4 samples/pixel
Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

16 samples/pixel
Ambient Occlusion

Uniform hemispherical sampling

Cosine-weighted importance sampling

1024 samples/pixel
Reflection Equation

- Which terms can we importance sample?

\[
L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i
\]

- BRDF
- incident radiance
- cosine term
Reflection Equation

- Which terms can we importance sample?

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

- BRDF
- incident radiance
- cosine term
Importance Sampling the BRDF

Cosine-weighted importance sampling

BRDF importance sampling

\[ p(\omega_i) \propto f(x, \omega_i, \omega_r) \]
Importance Sampling the BRDF

Uniform hemispherical sampling

BRDF importance sampling
Importance Sampling the BRDF

Recipe:

1. Find a convenient coordinate frame for drawing samples from the desired distribution
   • Compute the Jacobian of the transform between the sampling frame and the frame where you integrate
2. Compute marginal and conditional 1D PDFs
3. Sample 1D PDFs using the inversion method
Example: Normalized Phong

- Normalized Phong-like $\cos^\alpha$ lobe:
Example: Normalized Phong

- Normalized Phong-like $\cos^\alpha$ lobe:

\[
p(\theta, \phi) = \frac{\alpha + 2}{2\pi} \cos^\alpha \theta
\]

\[
(\theta, \phi) = \left( \cos^{-1} \left( (1 - \xi_1) \frac{1}{\alpha+2} \right), 2\pi \xi_2 \right)
\]
Example: Normalized Phong

- Normalized Phong-like $\cos^\alpha$ lobe:

$$p(\theta, \phi) = \frac{\alpha + 2}{2\pi} \cos^\alpha \theta$$

$$\left(\theta, \phi\right) = \cos^{-1} \left( \left(1 - \xi_1 \frac{1}{\alpha+2}\right), 2\pi \xi_2 \right)$$
Example: Normalized Phong

- Normalized Phong-like $\cos^\alpha$ lobe:

\[
p(\theta, \phi) = \frac{\alpha + 2}{2\pi} \cos^\alpha \theta
\]

\[
(\theta, \phi) = \left( \cos^{-1} \left( (1 - \xi_1) \frac{1}{\alpha+2} \right), 2\pi \xi_2 \right)
\]

Caution: $\theta, \phi$ here are w.r.t. to the reflected direction (not w.r.t. the normal)
Example: Microfacet Models
Example: Microfacet Models

- Importance sampling of *microfacet* distribution
Example: Microfacet Models

• Importance sampling of *microfacet* distribution
  – sample the distribution to get a microfacet normal
Example: Microfacet Models

• Importance sampling of *microfacet* distribution
  – sample the distribution to get a microfacet normal
  – reflect the incident direction about the normal
BRDFs with Multiple Lobes
BRDFs with Multiple Lobes
BRDFs with Multiple Lobes

• Typically, each lobe has a scaling coefficient
BRDFs with Multiple Lobes

• Typically, each lobe has a scaling coefficient
  – $k_d$ diffuse coefficient
  – $k_s$ specular/glossy coefficient
BRDFs with Multiple Lobes

- Typically, each lobe has a scaling coefficient
  - $k_d$ diffuse coefficient
  - $k_s$ specular/glossy coefficient

- Importance sampling:
BRDFs with Multiple Lobes

• Typically, each lobe has a scaling coefficient
  – $k_d$ diffuse coefficient
  – $k_s$ specular/glossy coefficient

• Importance sampling:
  – Probabilistically choose a lobe, e.g. proportional to the coefficient
BRDFs with Multiple Lobes

• Typically, each lobe has a scaling coefficient
  – $k_d$ diffuse coefficient
  – $k_s$ specular/glossy coefficient

• Importance sampling:
  – Probabilistically choose a lobe, e.g. proportional to the coefficient
  – Sample a direction using the lobe
BRDFs with Multiple Lobes

• Typically, each lobe has a scaling coefficient
  – \( k_d \) diffuse coefficient
  – \( k_s \) specular/glossy coefficient

• Importance sampling:
  – Probabilistically choose a lobe, e.g. proportional to the coefficient
  – Sample a direction using the lobe

\[
p_\Omega(\omega) = p_1(l)p_\Omega(\omega|l)
\]
Reflection Equation

• Which terms can we importance sample?

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

– BRDF
– incident radiance
– cosine term
Reflection Equation

- Which terms can we importance sample?

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

- BRDF
- incident radiance
- cosine term
Importance Sampling Incident Radiance
Importance Sampling Incident Radiance

• Impossible without prior knowledge [Müller et al. 2017]
Importance Sampling Incident Radiance

• Impossible without prior knowledge [Müller et al. 2017]
• But for direct illumination we can explicitly sample emissive surfaces
Importance Sampling Emissive Surfaces

• Impossible without prior knowledge [Müller et al. 2017]

• But for direct illumination we can explicitly sample emissive surfaces

• Requires the area form of the reflection equation:

\[
L_r(x, z) = \int_{A_e} f_r(x, y, z) L_e(y, x) V(x, y) \frac{|\cos \theta_i| |\cos \theta_o|}{\|x - y\|^2} dA(y)
\]

Integrate over emissive surfaces only
Reflection Equation
Reflection Equation

• Which terms can we importance sample?

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

– BRDF
– incident radiance (direct only)
– cosine term
Reflection Equation

• Which terms can we importance sample?

\[ L_r(x, \omega_r) = \int_{H^2} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i \, d\omega_i \]

– BRDF
– incident radiance (direct only)
– cosine term

– Which terms **should** we importance sample?
Reflection Equation

- Which terms can we importance sample?

\[ L_r(x, \bar{\omega}_r) = \int_{H^2} f_r(x, \bar{\omega}_i, \bar{\omega}_r) L_i(x, \bar{\omega}_i) \cos \theta_i \, d\bar{\omega}_i \]

- BRDF
- incident radiance (direct only)
- cosine term

- Which terms **should** we importance sample?
  - depends on the context, hard to generalize…
Multiple Strategies
Multiple Strategies

Cosine-weighted hemisphere
Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area
Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area

A few samples wasted, pdf \sim contribution (good)
Multiple Strategies

Cosine-weighted hemisphere

A few samples wasted, pdf ~ contribution (good)

Uniform surface area

Most samples wasted (bad)
Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area

A few samples wasted, pdf ~ contribution (good)

Most samples wasted (bad)

pdf \propto\text{contribution} (bad)
Multiple Strategies

Cosine-weighted hemisphere

- A few samples wasted, pdf ~ contribution (good)
- Most samples wasted (bad)

Uniform surface area

- No samples wasted
- pdf ~ contribution (good)
Combining Multiple Strategies
Combining Multiple Strategies

• Could just average two different estimators:

\[
\frac{0.5}{N_1} \sum_{i=1}^{N_1} \frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2} \sum_{i=1}^{N_2} \frac{f(x_i)}{p_2(x_i)}
\]
Combining Multiple Strategies

• Could just average two different estimators:

\[
\frac{0.5}{N_1} \sum_{i=1}^{N_1} \frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2} \sum_{i=1}^{N_2} \frac{f(x_i)}{p_2(x_i)}
\]

– doesn’t really help: variance is additive
Combining Multiple Strategies

• Could just average two different estimators:

\[
\frac{0.5}{N_1} \sum_{i=1}^{N_1} \frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2} \sum_{i=1}^{N_2} \frac{f(x_i)}{p_2(x_i)}
\]

– doesn’t really help: variance is additive
Combining Multiple Strategies

• Could just average two different estimators:

\[
\frac{0.5}{N_1} \sum_{i=1}^{N_1} \frac{f(x_i)}{p_1(x_i)} + \frac{0.5}{N_2} \sum_{i=1}^{N_2} \frac{f(x_i)}{p_2(x_i)}
\]

– doesn’t really help: variance is additive

• Instead, sample from the average PDF

\[
\frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{0.5(p_1(x_i) + p_2(x_i))}
\]
Combining Multiple Strategies

Cosine-weighted hemisphere

Uniform surface area
Combining Multiple Strategies

Cosine-weighted hemisphere

\[ p_1(\vec{\omega}) = \frac{\cos \theta}{\pi} \]

Uniform surface area

\[ p_2(x) = \frac{1}{A} \]
Combining Multiple Strategies

Cosine-weighted hemisphere

\[ p_1(\vec{\omega}) = \frac{\cos \theta}{\pi} \]

Uniform surface area

\[ p_2(x) = \frac{1}{A} \quad p_2(\vec{\omega}) = \frac{1}{A} \frac{d^2}{\cos \theta} \]
Combining Multiple Strategies

\[ \frac{1}{N} \sum_{i=1}^{N} \frac{f(x_i)}{0.5(p_1(x_i) + p_2(x_i))} \]

- You are given two sampling functions and their corresponding pdfs:
  - float sample1(float rnd); float pdf1(float x);
  - float sample2(float rnd); float pdf2(float x);

- Create a new function:
  - float sampleAvg(float rnd);

- which has the corresponding pdf:
  - float pdfAvg(float x)
    
    \[
    \text{return } 0.5 * (\text{pdf1}(x) + \text{pdf2}(x))\]


Sample from Average PDF

float sampleAvg(float rnd)
{
    if (rand.nextFloat() < 0.5)
        return sample1(rnd);
    else
        return sample2(rnd);
}
Sample from Average PDF

```c
float sampleAvg(float rnd)
{
    if (rand.nextFloat() < 0.5)
        return sample1(rnd);
    else
        return sample2(rnd);
}
```

Requires extra random number (can be avoided)
Sample from Average PDF

```cpp
float sampleAvg(float rnd)
{
    if (rnd < 0.5)
        return sample1(2.0 * rnd);
    else
        return sample2(2.0 * rnd - 1.0));
}
```
Cosine-weighted Hemisphere
Uniform Surface Area
Average PDF
Visual Break

source: onebigphoto.com