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## Feature Extraction

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### What are features?

Features are inherent properties of data, independent of coordinate frames etc.

Dimension of a feature:

- 0: **point feature** (often defined by  $n$  equations for  $n$  coordinates)
- 1: **line-like feature** ( $n-1$  equations)
- 2: **surface-like feature**
- etc.
- $n$ : **region-type feature** (typically defined by a single inequality)

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### Region-type features

A feature is often indicated by high or low values of a **derived field**.  
Example: **vortical regions** in a flow field have been defined by

- large magnitude of **vorticity**  $\boldsymbol{\omega}(\mathbf{x}) = \nabla \times \mathbf{v}(\mathbf{x})$
- high absolute **helicity**  $\boldsymbol{\omega}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x})$  or **normalized helicity**  $\frac{\boldsymbol{\omega}(\mathbf{x}) \cdot \mathbf{v}(\mathbf{x})}{\|\boldsymbol{\omega}(\mathbf{x})\| \|\mathbf{v}(\mathbf{x})\|}$
- positive **pressure Laplacian**  $\nabla \cdot \nabla p(\mathbf{x})$
- positive **second invariant** of the velocity gradient  $\nabla \mathbf{v}(\mathbf{x})$
- two negative **eigenvalues** of  $\frac{\nabla \mathbf{v}(\mathbf{x})^2 + (\nabla \mathbf{v}(\mathbf{x})^T)^2}{2}$

The latter three definitions are parameter-free (preferred in feature definitions).

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### Point features in scalar fields

Point features in scalar fields:

- **local minima/maxima**
  - **saddle points**
- occur at zero gradient  $\nabla s(\mathbf{x}) = \mathbf{0}$  ( $n$  scalar equations),  
(places where height field is horizontal).

The above point features are the places where the contour line or isosurface changes its topology when the level is varied from min to max.

The **contour tree** (or **Reeb graph**) describes the split and join events.

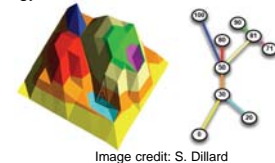


Image credit: S. Dillard

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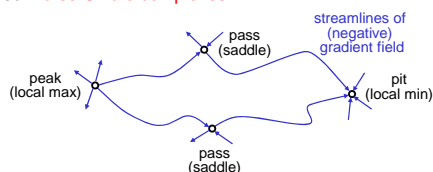
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### Line-like features in scalar fields

Line-like features in 2D scalar field:

**Watersheds** describe ridges/valleys of a height field  $s(\mathbf{x})$ :  
integrate the **gradient field**  $\nabla s(\mathbf{x})$  (backward/forward),  
starting at **saddle points**.

The watersheds provide a **segmentation** of the domain into so-called **Morse-Smale complexes**.



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### Line-like features in scalar fields

Watersheds require integration, are therefore not locally detectable.

Alternative definition of ridges/valleys (in  $nD$  scalar fields)?

Local minima/maxima:

- Known at least since differentiation was invented (17<sup>th</sup> century)!
- What is the natural extension to 1D?

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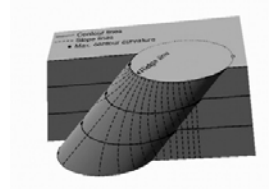
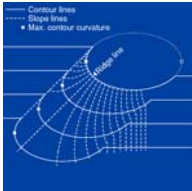
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Line-like features in scalar fields

Often used concepts:

- profile-based ridges
- curvature extrema on height contours

Counter-example for both (Wiener 1887!): "inclined elliptic cylinder"



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Line-like features in scalar fields

Question: How are local maxima most naturally extended to 1D features?

Answer: **height ridges**.

Surprisingly, a formal definition of height ridges was given only in the 1990s (Eberly, Lindeberg), based on Haralick's definition (1983).

In contrast, local minima/maxima are known for centuries.

De Saint-Venant (1852) defined a concept similar to height ridges.

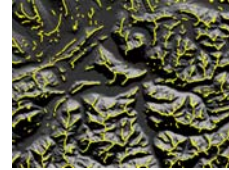


Image credit: P. Majer

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Line-like features in scalar fields

At a given point  $\mathbf{x}_0$  the scalar field has the Taylor approximation

$$s(\mathbf{x}_0 + \mathbf{x}) = s(\mathbf{x}_0) + \nabla s \cdot \mathbf{x} + \mathbf{x}^T \mathbf{H} \mathbf{x} + O(|\mathbf{x}|^3)$$

where  $\mathbf{H}$  is the **Hessian** matrix of second derivatives

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 s(\mathbf{x})}{\partial x_i \partial x_j} \end{pmatrix}$$

$\mathbf{H}$  has real eigenvalues and orthogonal eigenvectors.

By taking the eigenvectors as the coordinate frame,  $\mathbf{H}$  becomes the diagonal matrix

$$\mathbf{H} = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$$

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Line-like features in scalar fields

A point  $\mathbf{x} \in \mathbb{R}^n$  is a **local maximum** of  $s(\mathbf{x})$  if for **all**  $n$  axes:

- the first derivatives are zero:  $s_{x_1} = \dots = s_{x_n} = 0$
- the second derivatives are negative:  $s_{x_i x_i}, \dots, s_{x_n x_n} < 0$

In the appropriate coordinate frame, this generalizes to:

A point  $\mathbf{x} \in \mathbb{R}^n$  is on a  $d$ -dimensional **height ridge** of  $s(\mathbf{x})$  if for the **first  $n-d$  axes**:

- first derivatives are zero:  $s_{x_1} = \dots = s_{x_{n-d}} = 0$
- second derivatives are negative:  $s_{x_i x_i}, \dots, s_{x_{n-d} x_{n-d}} < 0$

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Line-like features in scalar fields

Appropriate coordinate frame means: axes are

- aligned with eigenvectors of  $\mathbf{H}$
- ordered by absolute eigenvalues:  $|\lambda_1| \geq \dots \geq |\lambda_n|$

Remark: We used Lindeberg's definition. In Eberly's definition axes are ordered by **signed** eigenvalues:  $\lambda_1 \leq \dots \leq \lambda_n$

This is slightly weaker (accepting more points).

Example: scalar field, (1D) height ridge according to Eberly and Lindeberg:



image credit: P. Majer

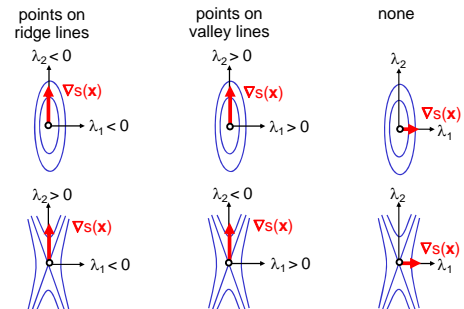
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Line-like features in scalar fields

Sketch of cases (with Lindeberg's definition,  $|\lambda_1| \geq |\lambda_2|$ )



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Line-like features in scalar fields

"Circular gutter" example (Koenderink / van Doorn):

Height field in polar coordinates:

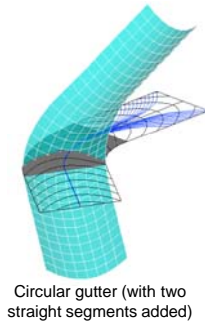
$$z(r, \varphi) = (1-r)^2 + k\varphi$$

- $k$  describes the steepness in the tangential direction.

- Profiles in radial sections are parabolas:

$$z(r, \varphi) = (1-r)^2 + \text{const}$$

- Lowest points in sections  $\varphi = \text{const}$  lie on the asymptote circle  $r=1$ .



Circular gutter (with two straight segments added)

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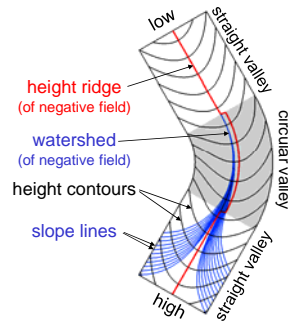
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Line-like features in scalar fields

Circular gutter:

Height ridge deviates (in the circular part) from the solution given by radial profiles.

"Counter-example" for height ridges.



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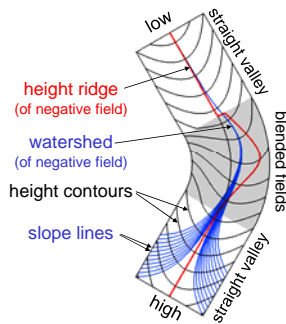
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Line-like features in scalar fields

Blended height fields (replacing the circular part by a blend of the two height fields):

Watershed deviates (in the lower part) from obvious symmetric valley line.

"Counter-example" for watersheds.



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Line-like features in scalar fields

The use of watersheds vs. height ridges is still heavily discussed in computer vision (Koenderink/van Doorn '93).

Watersheds:

- + are **slope lines** of height field (=streamlines of gradient field)
- depend on boundaries
- require existence of a saddle point

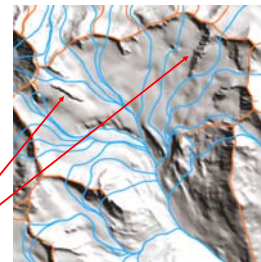
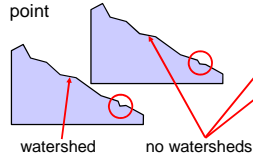


Image credit: C. Steger

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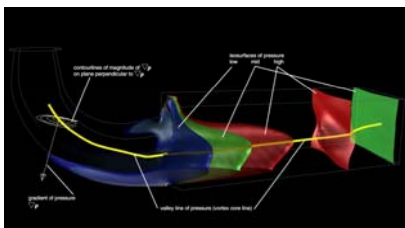
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Line-like features in scalar fields

Height ridges in 3D scalar fields can be used for defining/detecting **vortex core lines**. These are

- by Kida and Miura: height ridges (valley lines) of **pressure**



- by Ahmad/Kenwright/Strawn: height ridges of **vorticity magnitude**

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Geometric features of surfaces

On **surfaces** in 3-space, 0- and 1-dimensional features can be defined by the (differential) geometry alone.

**Geometric features vs. features of a field.**

Examples of geometric features (not a core subject of SciVis), based on **principal curvatures**  $\kappa_1, \kappa_2, |\kappa_1| \geq |\kappa_2|$

- **umbilic points:**  $\kappa_1 = \kappa_2$
- **curvature ridges:** Loci of points where  $|\kappa_1|$  is a maximum along the associated curvature line



Image credit: Y. Ohtake

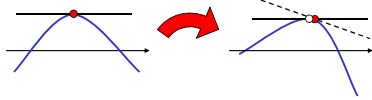
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Geometric features of surfaces

The term "ridge" can refer to either height ridges or curvature ridges.  
 Curvature ridges are not appropriate as features of a scalar field (height field).  
 Reason: Invariance under rotation (tilting).



Line-like features in vector fields

Height ridges of a scalar field  $s(\mathbf{x})$  are definable by the gradient field  $\mathbf{v}(\mathbf{x}) = \nabla s(\mathbf{x})$  alone:

- $\mathbf{H}$  is its Jacobian  $\nabla \mathbf{v}(\mathbf{x})$ , and
- $s(\mathbf{x})$  itself is not needed.

A necessary condition for a height ridge is:

$$\mathbf{v}(\mathbf{x}) \text{ is an eigenvector of } \nabla \mathbf{v}(\mathbf{x})$$

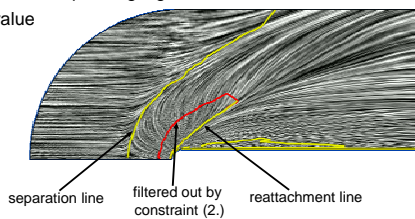
The gradient is a **conservative** (irrotational) vector field.  
 Let's now extend this to **general** vector fields.

Line-like features in vector fields

**Separation lines** in boundary shear flow (Kenwright):

A point  $\mathbf{x}$  lies on a **separation** or **reattachment line** if

1. at  $\mathbf{x}$  the field vector  $\mathbf{v}(\mathbf{x})$  is an eigenvector of  $\nabla \mathbf{v}(\mathbf{x})$
2. the corresponding eigenvalue is the one with smaller absolute value



Line-like features in vector fields

**Vortex core lines** (Sujudi / Haimes):

A point  $\mathbf{x}$  lies on a **vortex core line** if

1. at  $\mathbf{x}$  the field vector  $\mathbf{v}(\mathbf{x})$  is an eigenvector of  $\nabla \mathbf{v}(\mathbf{x})$
2. the two other eigenvalues of  $\nabla \mathbf{v}(\mathbf{x})$  are complex

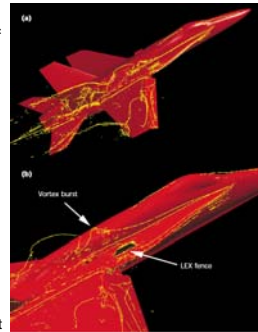


Image credit: D. Kenwright

Line-like features in vector fields

Alternative definitions of vortex core lines:

- According to Levy et al., longitudinal vortices have high normalized helicity (or small angles between velocity and vorticity).  
 → vortex core line criterion:  $\mathbf{v}(\mathbf{x})$  is (anti-) parallel to  $\boldsymbol{\omega}(\mathbf{x})$ .

- Singer and Banks' method:
  - find a first point on the core line
  - repeat
    - predict next point along  $\boldsymbol{\omega}(\mathbf{x})$
    - correct to pressure minimum in normal plane of  $\boldsymbol{\omega}(\mathbf{x})$
  - compute vortex hull

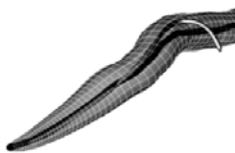
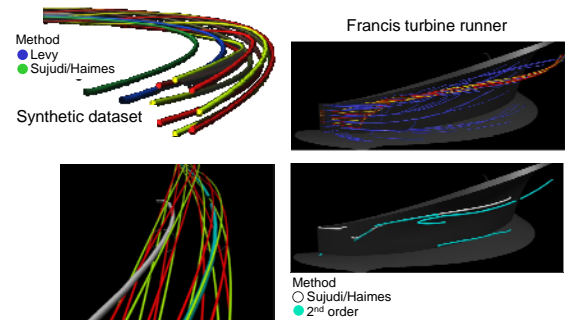


Image credit: D. Banks

Line-like features in vector fields

Deviation of locally computed vortex core lines:



Line-like features in vector fields

Discussion: local vs. global features  
 Global features: e.g. streamlines.  
 Are vortex core lines streamlines?

Here is a "counter-example":



Line-like features in vector fields

How to compute line-like features?  
 Instead of explicitly computing eigenvectors for height ridges, Sujudi-Haimes core lines, etc.:

Make use of observation:

$\mathbf{v}$  is eigenvector of  $\mathbf{A}$  if and only if  $\mathbf{A}\mathbf{v}$  is **parallel** to  $\mathbf{v}$  (because  $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$ )

Recipe:

- compute  $\mathbf{w} = \mathbf{A}\mathbf{v}$  as a **derived field**
- find places where  $\mathbf{v}$  and  $\mathbf{w}$  are parallel (or one of them is 0).
- apply constraints
- apply post-filtering (vortex strength, etc.)

Line-like features in vector fields

Parallel vectors operator:  
 given  $\mathbf{v}, \mathbf{w}$ : returns points where  $\mathbf{v}$  and  $\mathbf{w}$  are parallel

Implementation:

- in 2D:  $\mathbf{v} \times \mathbf{w} = 0$  is just a contour line problem
- In 3D:  $\mathbf{v} \times \mathbf{w} = 0$  is 3 equations for 3 unknowns:
  - equations are linearly dependent
  - can be solved with Marching Cubes like method

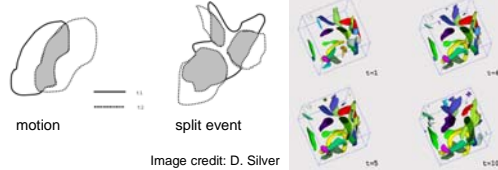
Tracking of features

In time-dependent data, features are usually extracted for single time steps.

How to recognize a feature in a different time step?

Some methods are:

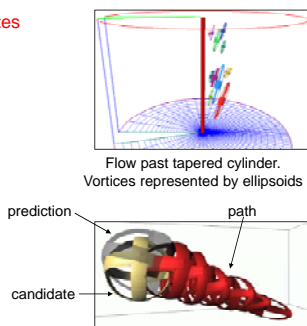
- Decide on **spatial overlap** (Silver et al.)
  - appropriate for region-type features
  - detects motion and events (split, merge, birth, death)



Tracking of features

- Decide on **feature attributes** (Reinders)

- use attributes such as position, shape (fitted ellipsoid), orientation, spin, data values, etc.
- combine with motion prediction



Tracking of features

- **Lift** the feature extraction method to **space-time domain**.

Examples:

- Critical points in 2-space + time (Tricoche):  
 Equations  $u(x, y, t) = 0, v(x, y, t) = 0$  yield **lines** when solved in an "extruded"  $(x, y, t)$  grid.  
 Features move along these lines, no explicit tracking needed.
- Vortex core lines in 3-space + time (Bauer):  
 Feature extraction yields a 2D mesh in 4-space.  
 Time-slice is a line-like feature.