Tensor Field Visualization

Tensor is the extension of concept of scalar and vector, it is the language of mechanics. Therefore, tensor field visualization is a challenging issue for scientific visualization. Scientists and engineers need techniques that enable both qualitative and quantitative analysis of tensor data sets resulting from experiments or numerical simulations. Tensor data for a tensor of level k is given by $t_{i1,i2,\ldots,ik}(x_{1,}\ldots,x_n)$, second-order tensor are often represented by a matrix.

Examples:

- Diffusion tensor (from medical imaging, see later)
- Material properties (material sciences):
- Conductivity tensor1
- Dielectric susceptibility
- Magnetic permutivity
- Stress tensor

1 Diffusion Tensor

A typical second-order tensor is the diffusion tensor. Its characteristics are:

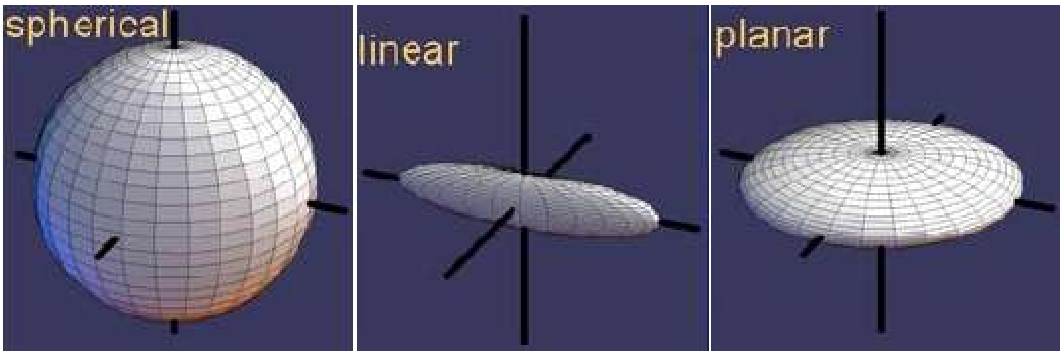
- Diffusion: based on motion of fluid particles on microscopic level
- Probabilistic phenomenon
- Based on particle’s Brownian motion
- Measurements by modern MR (magnetic resonance) scanners
- Diffusion tensor describes diffusion rate into different directions via symmetric tensor (probability density distribution)
- In 3D: representation via $3*3$ symmetric matrix

Symmetric diffusion matrix can be diagonalized:

- 3 Real eigenvalues $\lambda_1 \geqslant \lambda_2 \geqslant \lambda_3$
- Eigenvectors are perpendicular

Isotropy / anisotropy:

- Spherical: $\lambda_1 = \lambda_2 = \lambda_3$
- Linear: $\lambda_2 \approx \lambda_1 \approx 0$
- Planar: $\lambda_1 \approx \lambda_2$ and $\lambda_1 \approx 0$



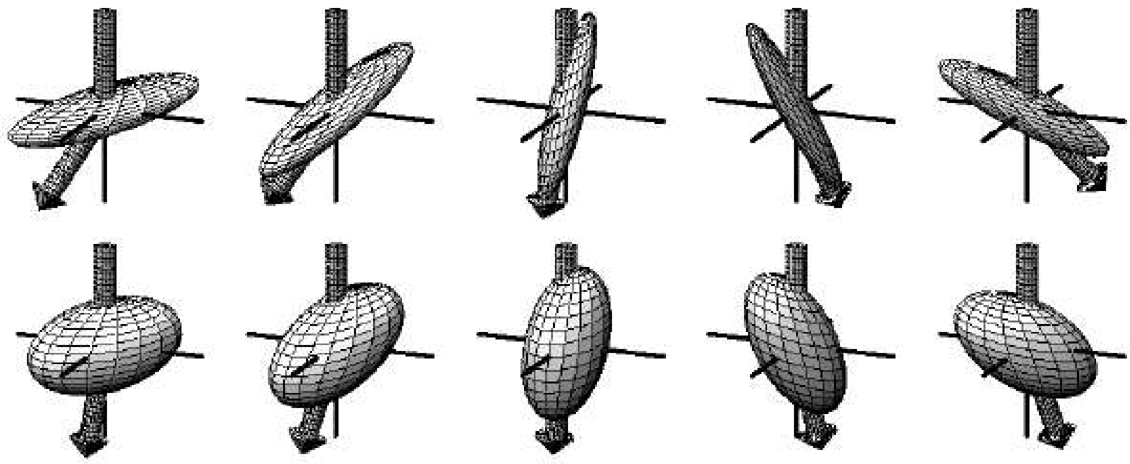
Spherical: all eigenvalues are the same.

Linear: One eigenvalue is large, the others are almost 0.

Planar: Two eigenvalues are large and one is almost 0.

- Arbitrary vectors are generally deflected after matrix multiplication

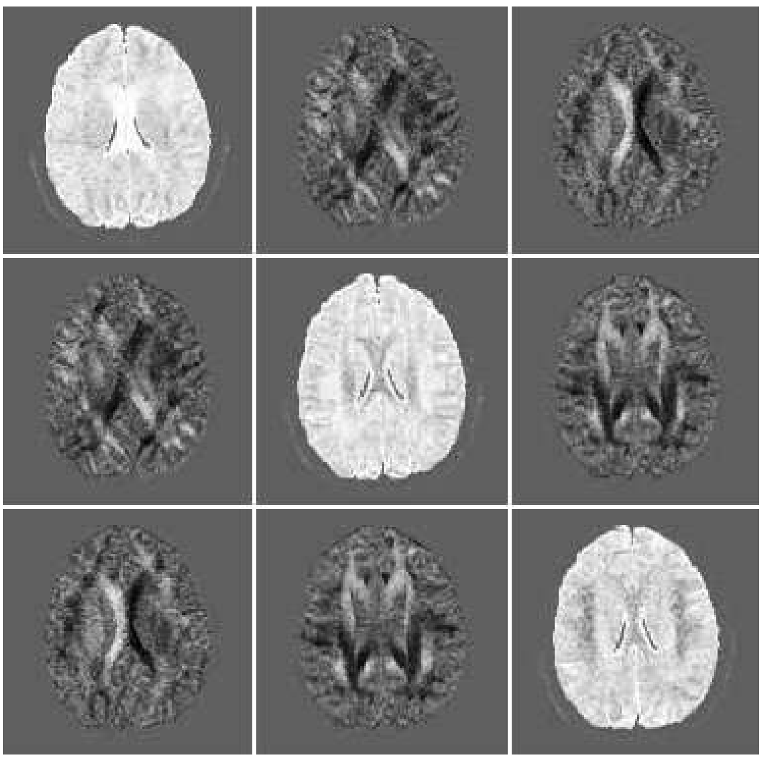
- Deflection into direction of principal eigenvector (largest eigenvalue)



Tensor is something that you apply to a vector and you get a different vector. So the Tensor tells you about the behavior of the vector.

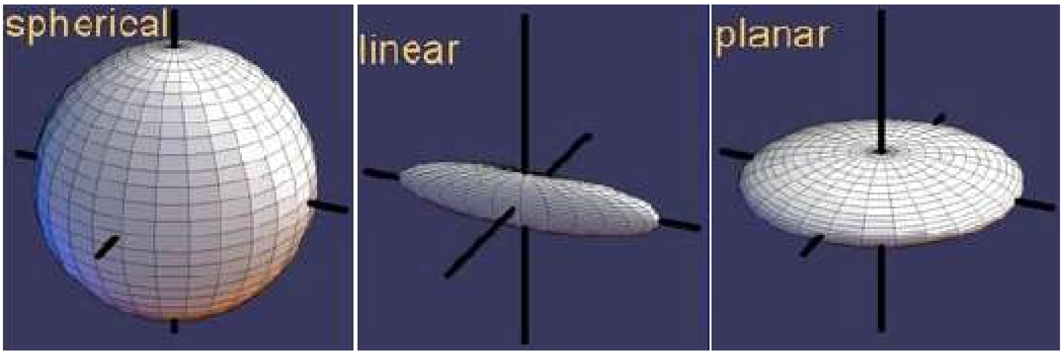
2 Basic Mapping Techniques

Matrix of images:
- Slices through volume
- Each image shows one component of the matrix



This illustration shows a matrix of images of diffusion tensor.

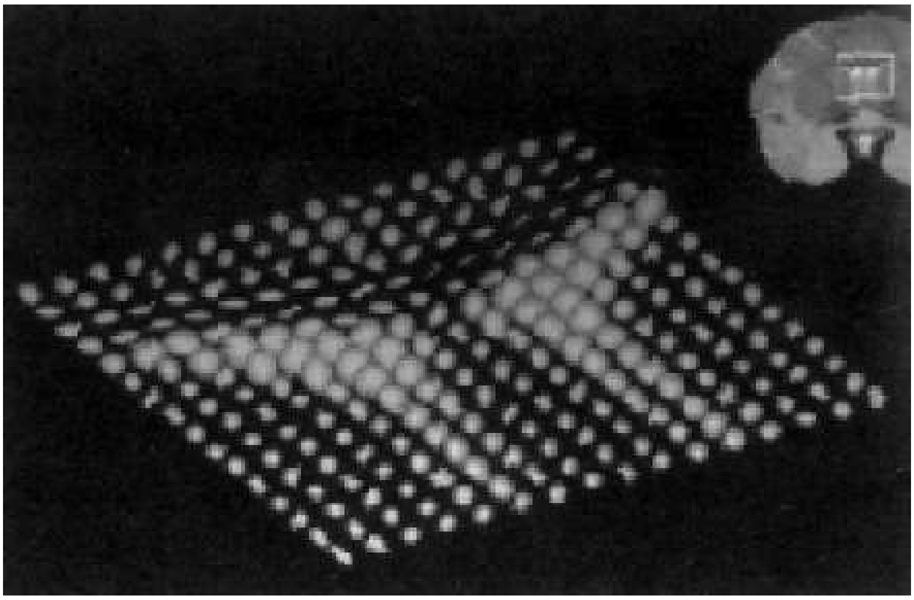
Symmetric tensor can be diagonalized
- Representation by an ellipsoid
- Glyph-based approach



Example for isotropy (spherical) and anisotropy.

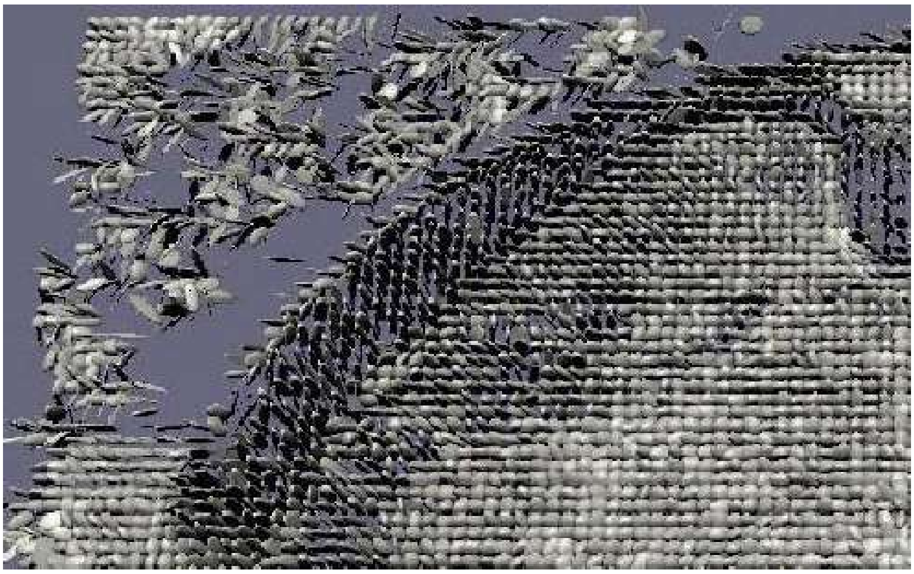
Diffusion tensor magnetic resonance imaging *[Pierpaoli-1996-MRI]*:
C. Pierpaoli, P. Jezzard, P. J. Basser, A. Barnett, and G. Di Chiro, Diffusion tensor MR imaging of the human brain, Radiology, vol. 201, no. 3, pp. 637--648, 1996

- Uniform grid of ellipsoids
- Second-order symmetric tensor mapped to ellipsoid
- Sliced volume



Example for a Diffusion Tensor Magnetic Resonance Image of the human brain.

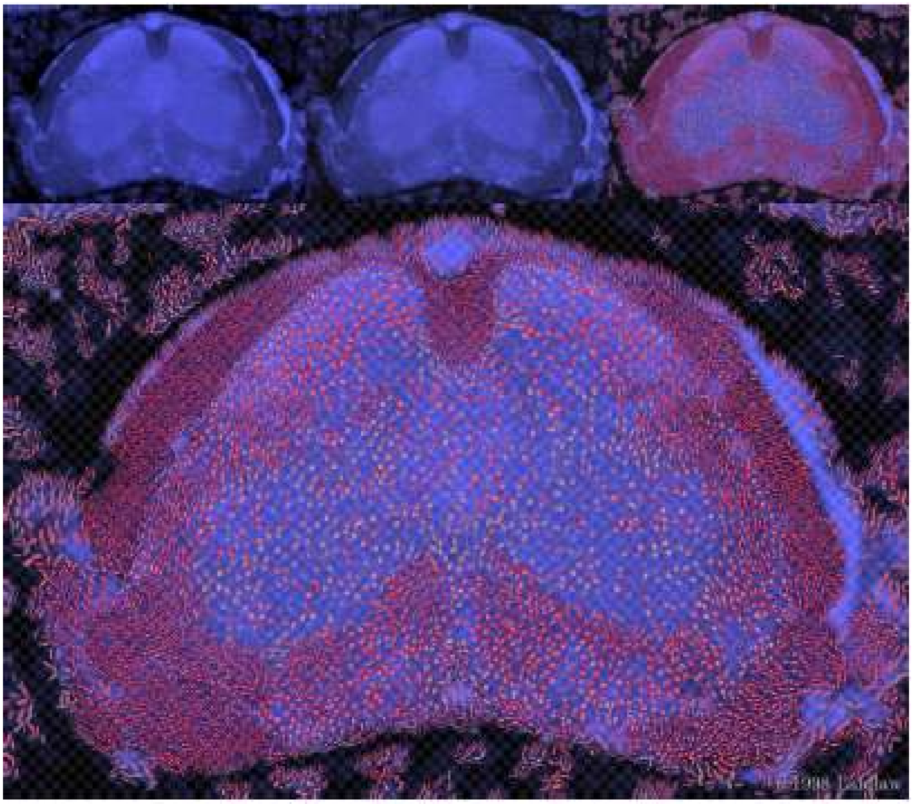
- Uniform grid of ellipsoids
- Normalized sizes of the ellipsoids



Brushstrokes *[Laidlaw-1998-VDT]*

- Influenced by paintings
- Multivalued data
- Scalar intensity
- Sampling rate
- Diffusion tensor
- Textured strokes

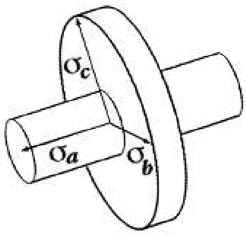
scalar *sampling rate* *tensor*

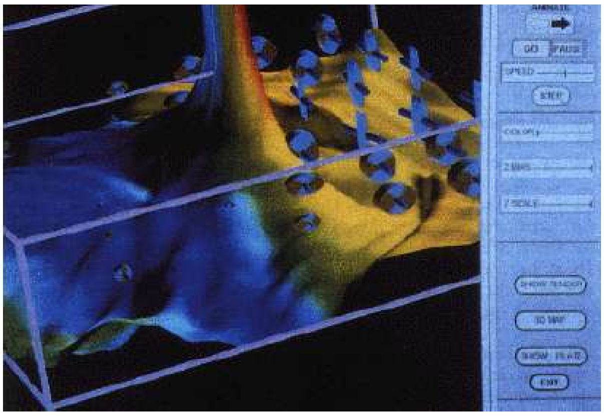


Ellipsoids in 3D:

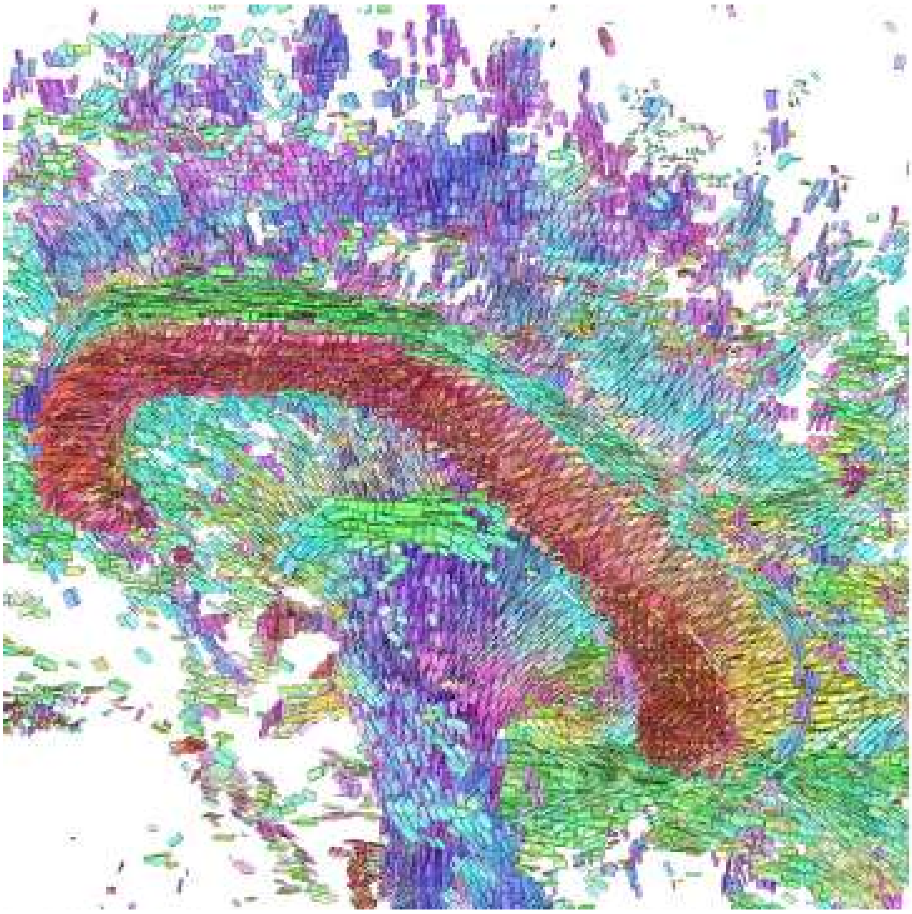
- Problems:
- Occlusion
- Missing continuity

Haber glyphs [Haber-1990]
- Rod and elliptical disc
- Better suited to visualize magnitudes of the tensor and principal axis

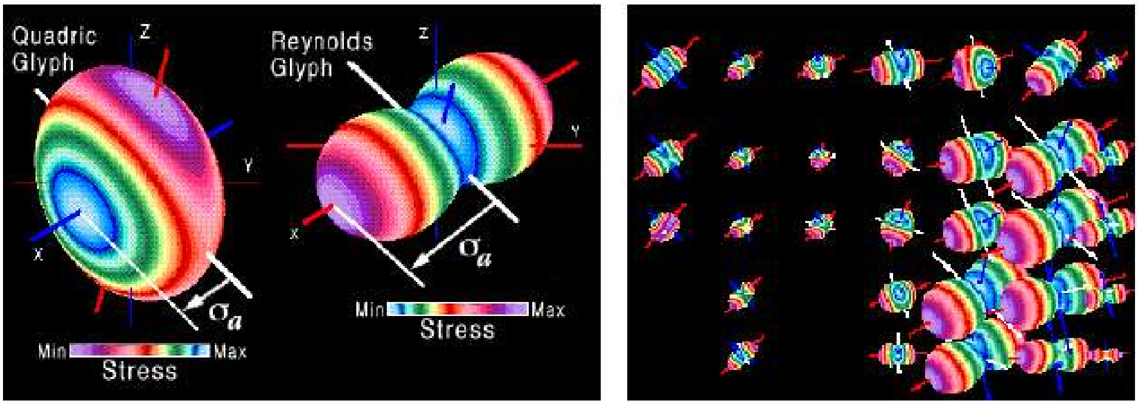




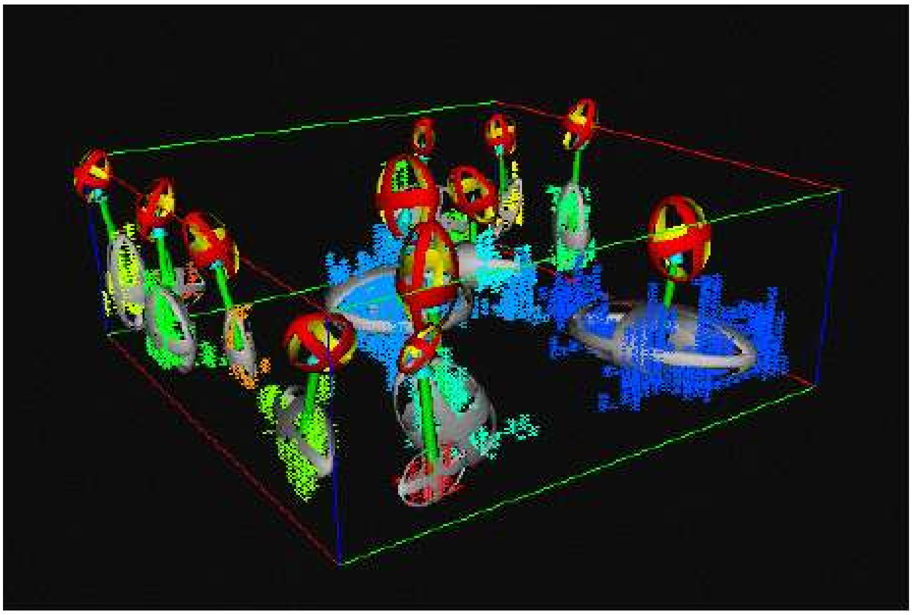
Box glyphs [Johnson-2001]



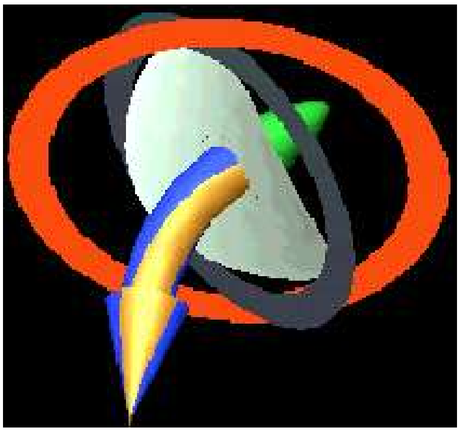
Reynolds glyph [Moore-1994]

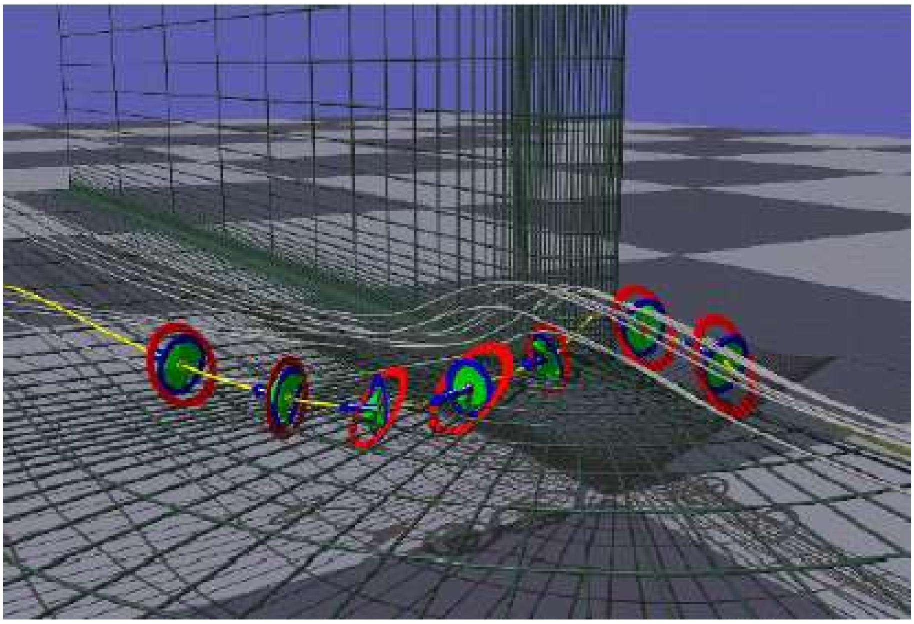


Generic iconic techniques for feature visualization [Post-1995]

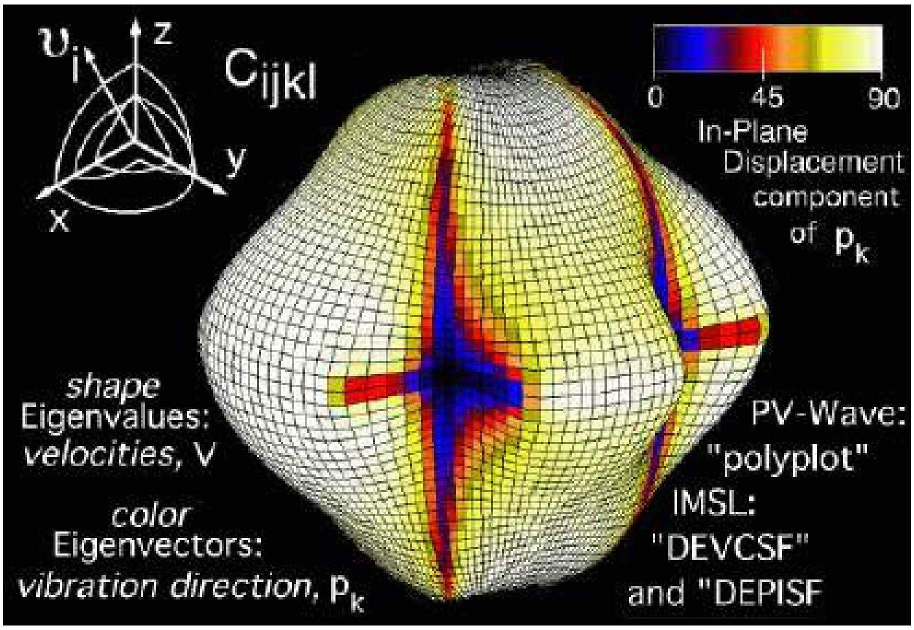


Glyph probe for local flow field visualization [Leeuw, Wijk 1993]
- Arrow: particle path
- Green cap: tangential acceleration
- Orange ring: shear (with respect to gray ring)





Glyph for fourth-order tensor
- (wave propagation in crystals)



3 Hue-Balls and Lit-Tensors

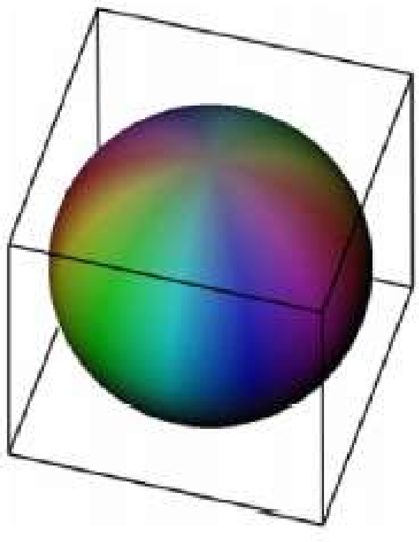
Hue-balls and **Lit-tensors** [Kindlmann, Weinstein 1999]

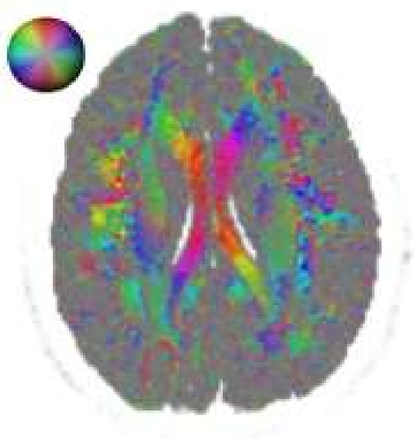
- Ideas and elements
- Visualize anisotropy (relevant, e.g., in biological applications)
- Color coding
- Opacity function
- Illumination
- Volume rendering

- Color coding (hue-ball)
- Fixed, yet arbitrary input vector (e.g., user specified)
- Color coding for output vector
- Coding on sphere

Idea:

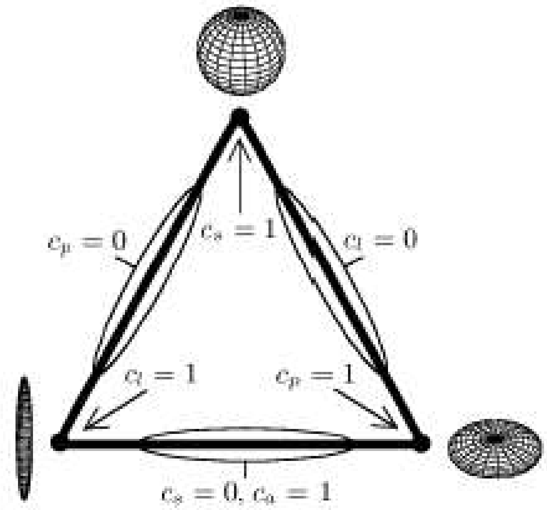
- Deflection is strongly coupled with anisotropy





Barycentric opacity mapping

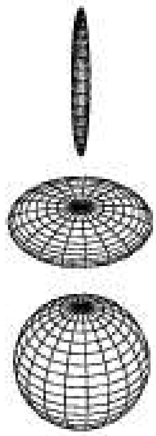
- Emphasize important features
- Make unimportant regions transparent
- Can define 3 barycentric coordinates cl, cp, cs

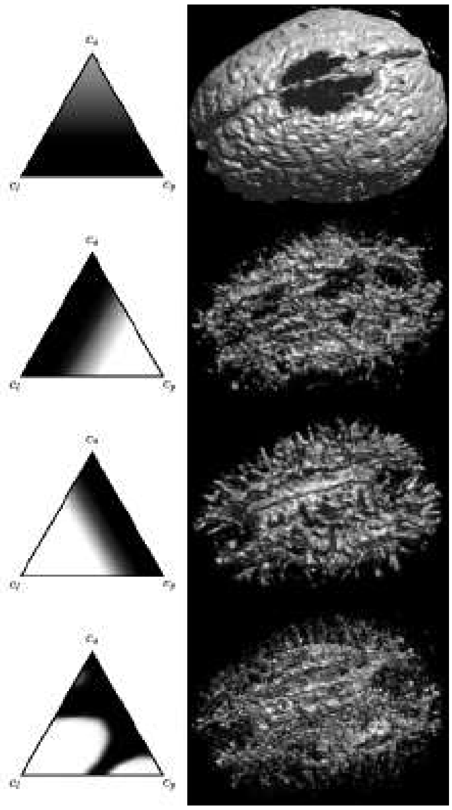


$$c_l = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$c_p = \frac{2(\lambda_2 - \lambda_3)}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$c_s = \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$



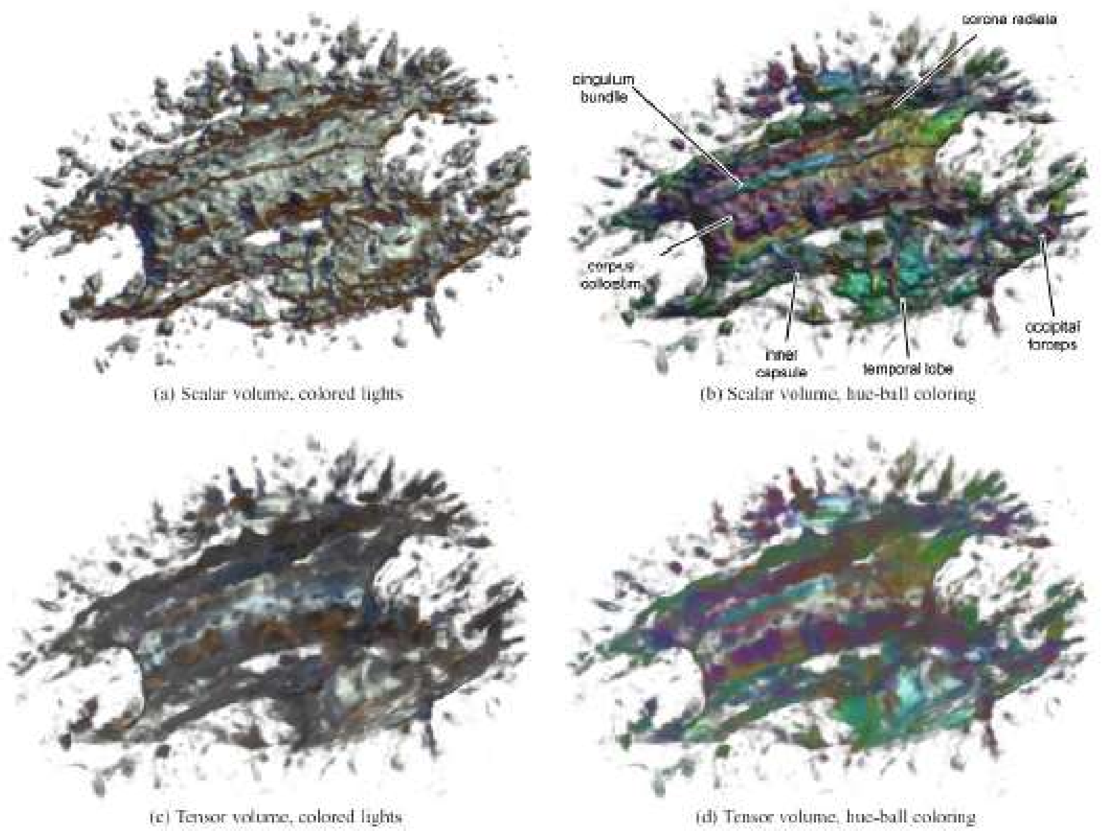


Examples for transfer functions.

Lit-tensors

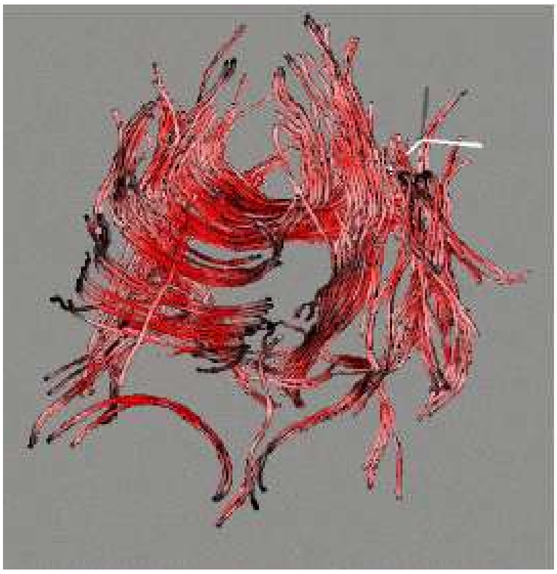
- Similar to illuminated streamlines
- Illumination of tensor representations
- Provide information on direction and curvature
- Cases
- Linear anisotropy: same as illuminated streamlines
- Planar anisotropy: surface shading
- Other cases: smooth interpolation between these two extremes

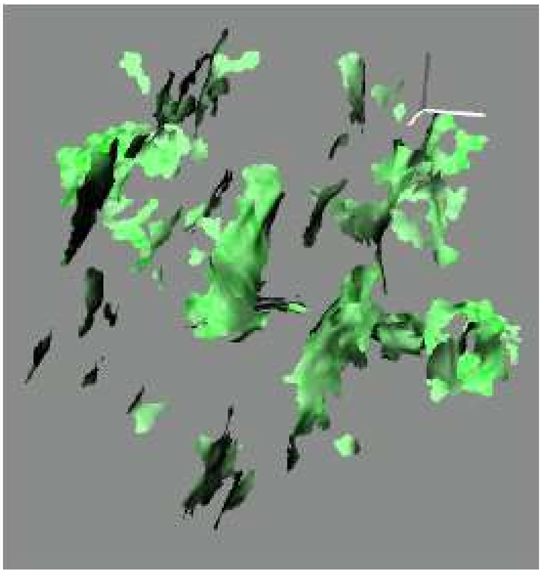
Example for different Lit-tensors.



(a) Scalar volume, colored lights
(b) Scalar volume, hue-ball coloring
(c) Tensor volume, colored lights
(d) Tensor volume, hue-ball coloring

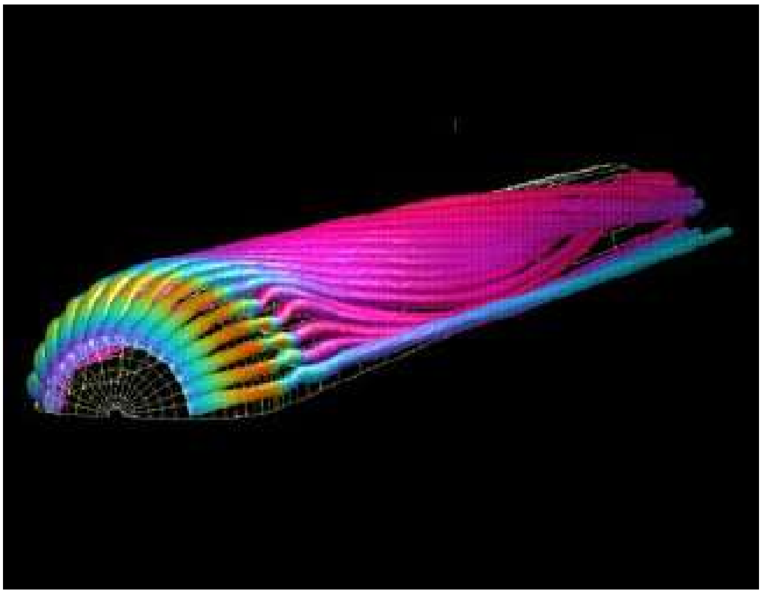
Variation: **streamtubes** and **streamsurfaces** [Zhang-2000]
- Streamtubes: linear anisotropic regions
- Streamsurfaces: planar anisotropic surfaces



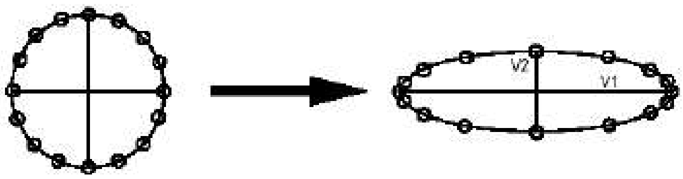


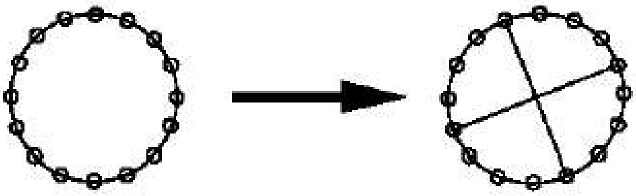
4 Hyperstreamlines and Tensorlines

Hyperstreamlines [Delmarcelle, Hesselink 1992/93]
- Streamlines defined by eigenvectors
- Direction of streamline by major eigenvector
- Visualization of the vector field defined by major eigenvector
- Other eigenvectors define cross-section

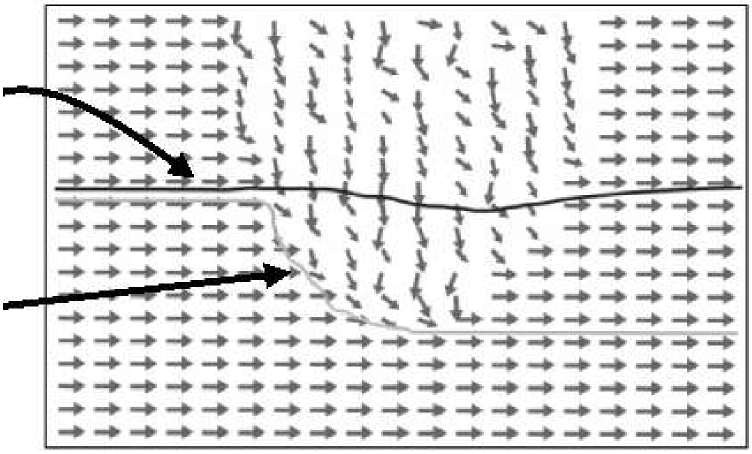


- Idea behind hyperstreamlines:
- Major eigenvector describes direction of diffusion with highest probability density
- Ambiguity for (nearly) isotropic case



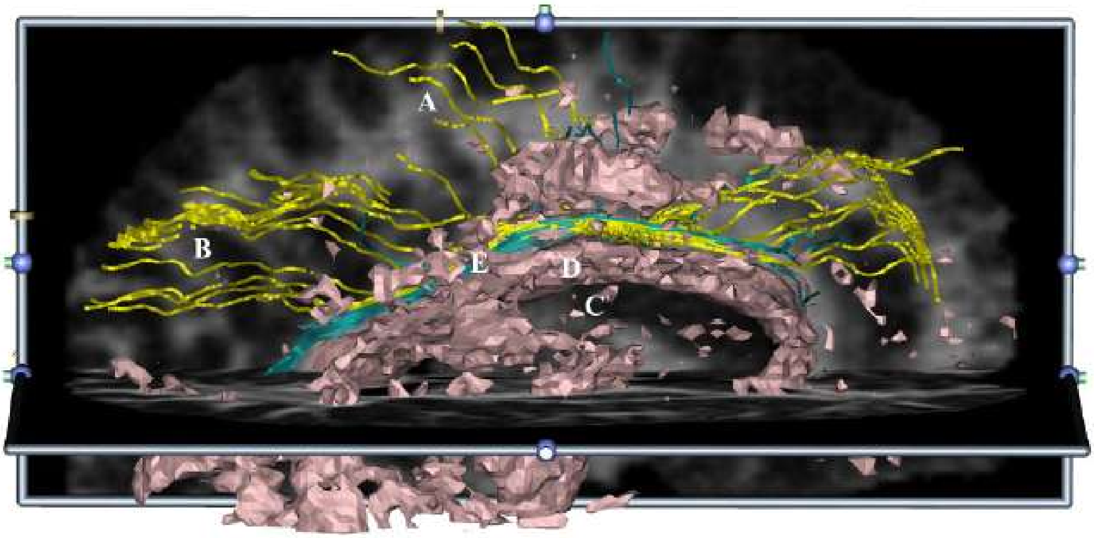


- Problems of hyperstreamlines
- Ambiguity in (nearly) isotropic regions:
- Partial voluming effect, especially in low resolution images (MR images)
- Noise in data
- Solution: tensorlines
- Tensorline
- Hyperstreamline
- Arrows: major eigenvector



Tensorlines [Weinstein, Kindlmann 1999]

- Advection vector
- Stabilization of propagation by considering
- Input velocity vector
- Output velocity vector (after application of tensor operation)
- Vector along major eigenvector
- Weighting of three components depends on anisotropy at specific position:
- Linear anisotropy: only along major eigenvector
- Other cases: input or output vector



Tensorlines.