8. Tensor Field Visualization

- Tensor: extension of concept of scalar and vector
- Tensor data
  for a tensor of level *k* is given by $t_{i1,i2,\ldots,ik}(x_1,\ldots,x_n)$
- Second-order tensor often represented by matrix

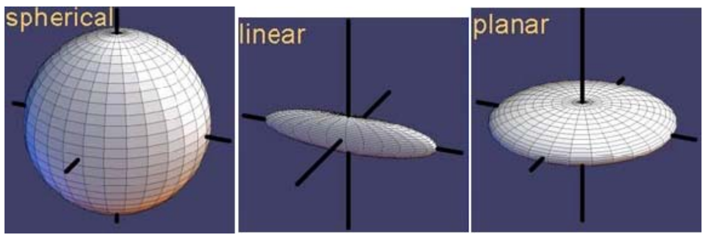
- Examples:
  - Diffusion tensor (from medical imaging, see later)
  - Material properties (material sciences):
    - Conductivity tensor
    - Dielectric susceptibility
    - Magnetic permutivity
    - Stress tensor

8.1. Diffusion Tensor

- Typical second-order tensor: diffusion tensor
  - Diffusion: based on motion of fluid particles on microscopic level
  - Probabilistic phenomenon
  - Based on particle's Brownian motion
  - Measurements by modern MR (magnetic resonance) scanners
  - Diffusion tensor describes diffusion rate into different directions via symmetric tensor (probability density distribution)
  - In 3D: representation via 3*3 symmetric matrix

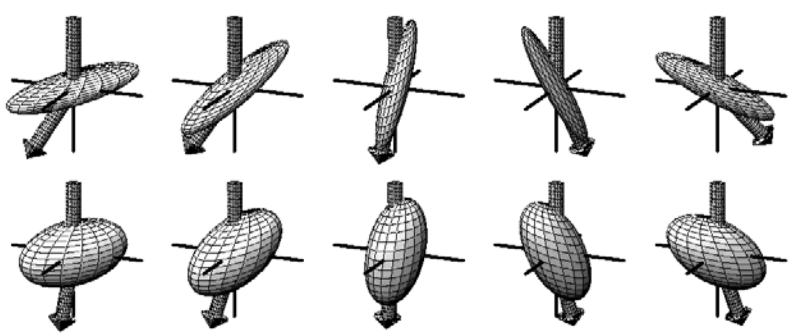
8.1. Diffusion Tensor

- Symmetric diffusion matrix can be diagonalized:
  - Real eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$
  - Eigenvectors are perpendicular
- Isotropy / anisotropy:
  - Spherical: $\lambda_1 = \lambda_2 = \lambda_3$
  - Linear: $\lambda_2 \approx \lambda_3 \approx 0$
  - Planar: $\lambda_1 \approx \lambda_2$ und $\lambda_3 \approx 0$



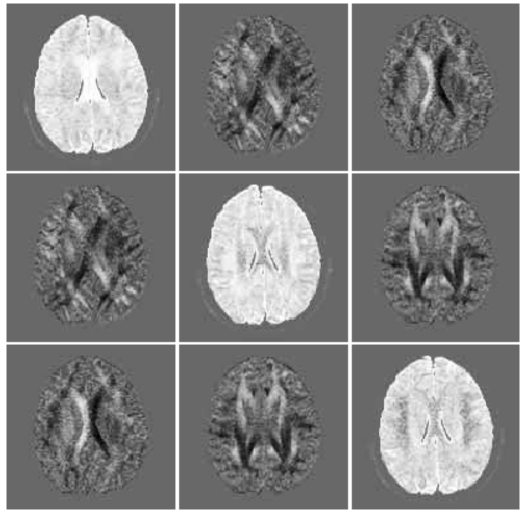
8.1. Diffusion Tensor

- Arbitrary vectors are generally deflected after matrix multiplication
- Deflection into direction of principal eigenvector (largest eigenvalue)



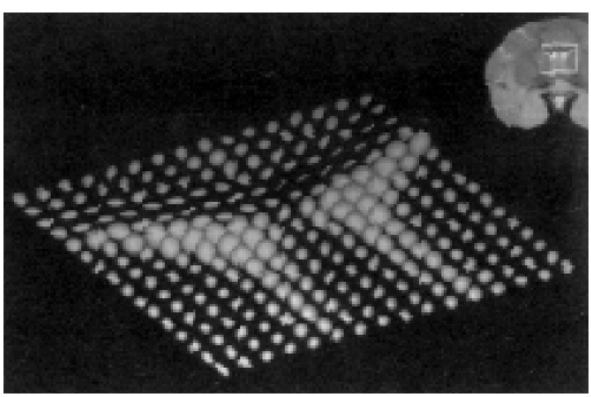
8.2. Basic Mapping Techniques

- Matrix of images
  - Slices through volume
  - Each image shows one component of the matrix



8.2. Basic Mapping Techniques

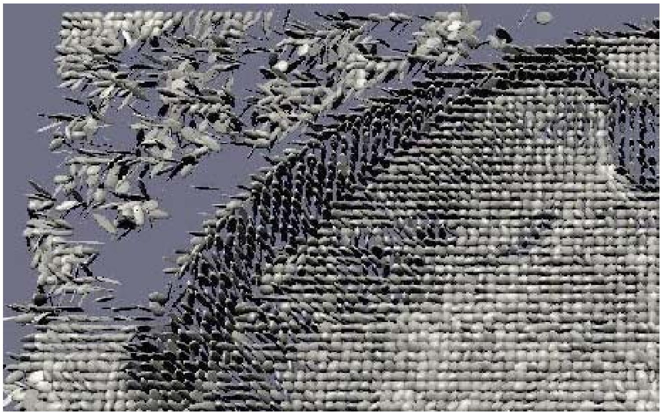
- Uniform grid of ellipsoids
  - Second-order symmetric tensor mapped to ellipsoid
  - Sliced volume



[Pierpaoli et al. 1996]

8.2. Basic Mapping Techniques

- Uniform grid of ellipsoids
  - Normalized sizes of the ellipsoids

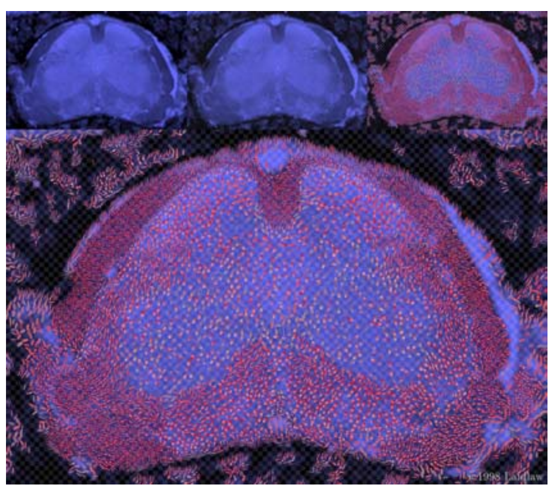


[Laidlaw et al. 1998]

8.2. Basic Mapping Techniques

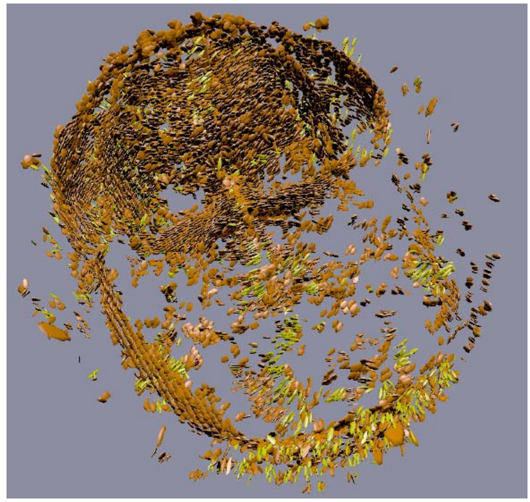
- Brushstrokes
  [Laidlaw et al. 1998]
  - Influenced by paintings
  - Multivalued data
  - Scalar intensity
  - Sampling rate
  - Diffusion tensor
  - Textured strokes

scalar sampling rate tensor



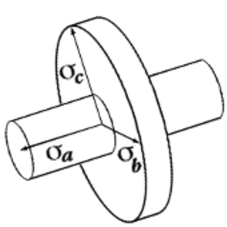
8.2. Basic Mapping Techniques

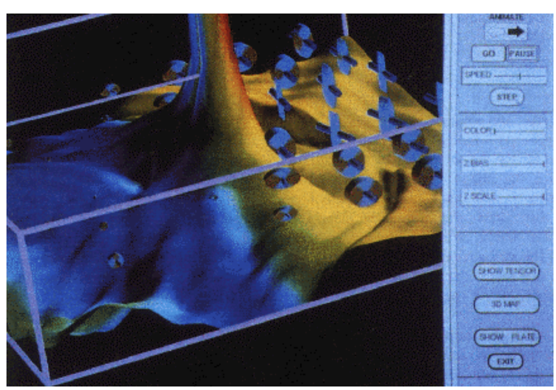
- Ellipsoids in 3D
- Problems:
  - Occlusion
  - Missing continuity



8.2. Basic Mapping Techniques

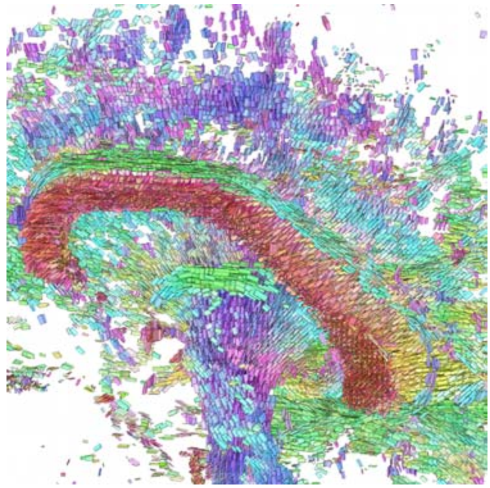
- Haber glyphs [Haber 1990]
  - Rod and elliptical disk
  - Better suited to visualize magnitudes of the tensor and principal axis





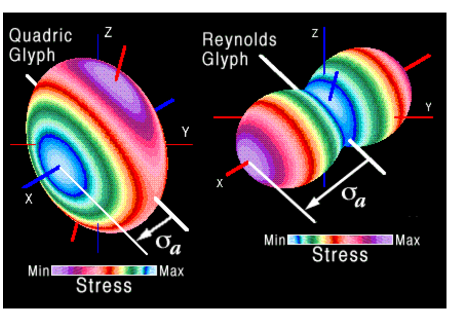
8.2. Basic Mapping Techniques

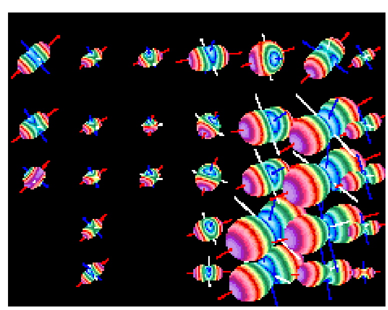
- Box glyphs
  [Johnson et al. 2001]



8.2. Basic Mapping Techniques

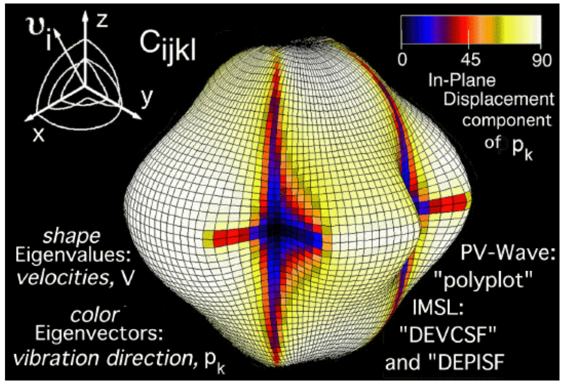
- Reynolds glyph [Moore et al. 1994]





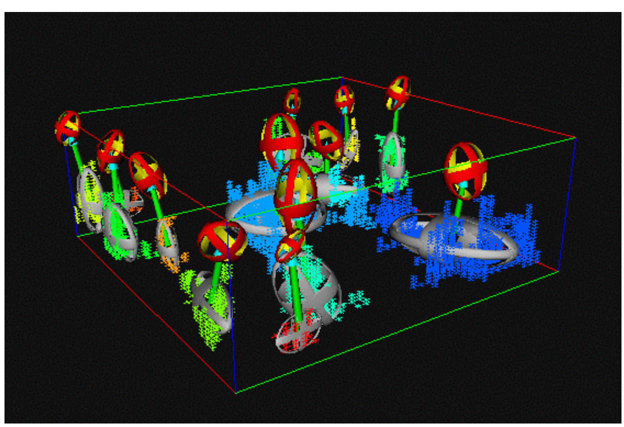
8.2. Basic Mapping Techniques

- Glyph for fourth-order tensor
  - (wave propagation in crystals)



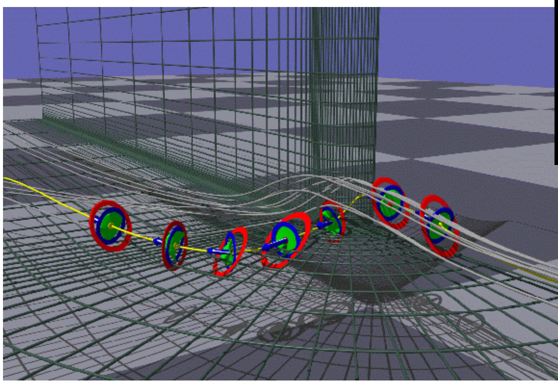
8.2. Basic Mapping Techniques

- Generic iconic techniques for feature visualization [Post et al. 1995]



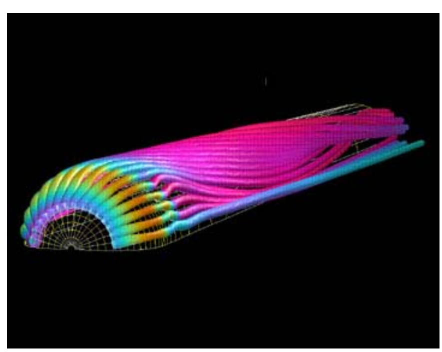
8.2. Basic Mapping Techniques

- Glyph probe for local flow field visualization [Leeuw, Wijk 1993]
  - Arrow: particle path
  - Green cap: tangential acceleration
  - Orange ring: shear (with respect to gray ring)



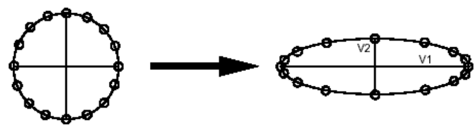
8.4. Hyperstreamlines and Tensorlines

- Hyperstreamlines [Delmarcelle, Hesselink 1992/93]
  - Streamlines defined by eigenvectors
  - Direction of streamline by major eigenvector
  - Visualization of the vector field defined by major eigenvector
  - Other eigenvectors define cross-section

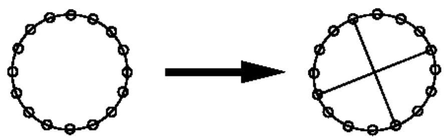


8.4. Hyperstreamlines and Tensorlines

- Idea behind hyperstreamlines:
  - Major eigenvector describes direction of diffusion with highest probability density

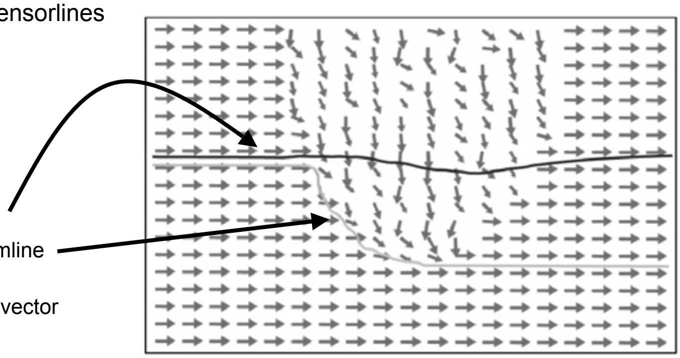


- Ambiguity for (nearly) isotropic case



8.4. Hyperstreamlines and Tensorlines

- Problems of hyperstreamlines
  - Ambiguity in (nearly) isotropic regions:
  - Partial voluming effect, especially in low resolution images (MR images)
  - Noise in data
  - Solution: tensorlines



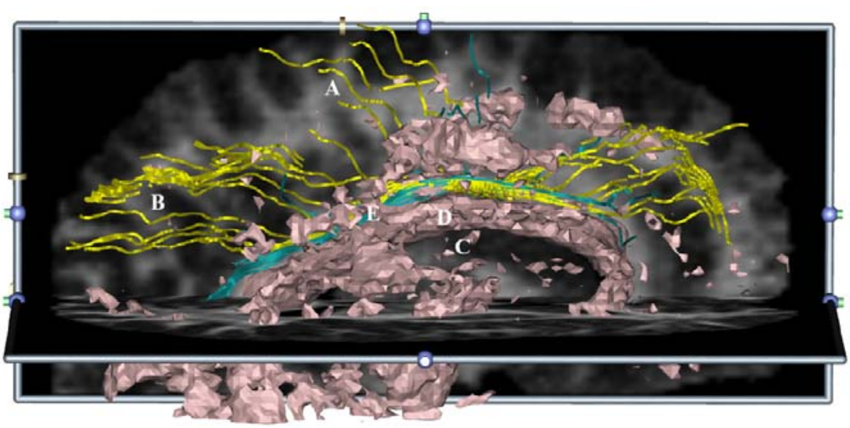
- Tensorline
- Hyperstreamline
- Arrows:
  major eigenvector

8.4. Hyperstreamlines and Tensorlines

- Tensorlines [Weinstein, Kindlmann 1999]
  - Advection vector
  - Stabilization of propagation by considering
    - Input velocity vector
    - Output velocity vector (after application of tensor operation)
    - Vector along major eigenvector
  - Weighting of three components depends on anisotropy at specific position:
    - Linear anisotropy: only along major eigenvector
    - Other cases: input or output vector

8.4. Hyperstreamlines and Tensorlines

- Tensorlines

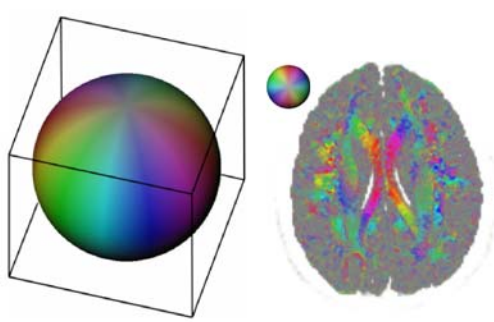


8.3. Hue-Balls and Lit-Tensors

- Hue-balls and lit-tensors [Kindlmann, Weinstein 1999]
- Ideas and elements
  - Visualize anisotropy (relevant, e.g., in biological applications)
  - Color coding
  - Opacity function
  - Illumination
  - Volume rendering

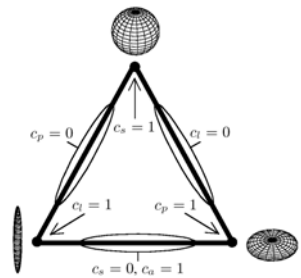
8.3. Hue-Balls and Lit-Tensors

- Color coding (hue-ball)
  - Fixed, yet arbitrary input vector (e.g., user specified)
  - Color coding for output vector
  - Coding on sphere
- Idea:
  - Deflection is strongly coupled with anisotropy



8.3. Hue-Balls and Lit-Tensors

- Barycentric opacity mapping
  - Emphasize important features
  - Make unimportant regions transparent
  - Can define 3 barycentric coordinates c_l, c_p, c_s



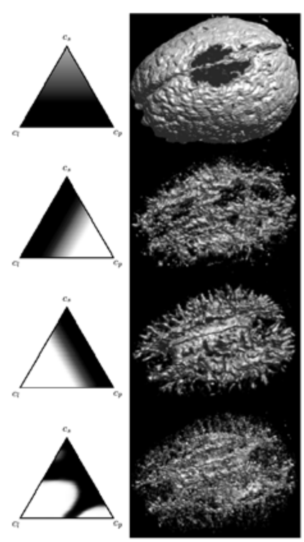
$$c_l = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$c_p = \frac{2(\lambda_2 - \lambda_3)}{\lambda_1 + \lambda_2 + \lambda_3}$$

$$c_s = \frac{3\lambda_3}{\lambda_1 + \lambda_2 + \lambda_3}$$

8.3. Hue-Balls and Lit-Tensors

- Barycentric opacity mapping *(cont.)*
  - Examples for transfer functions

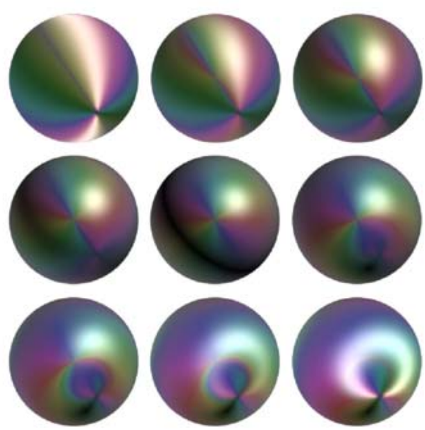


8.3. Hue-Balls and Lit-Tensors

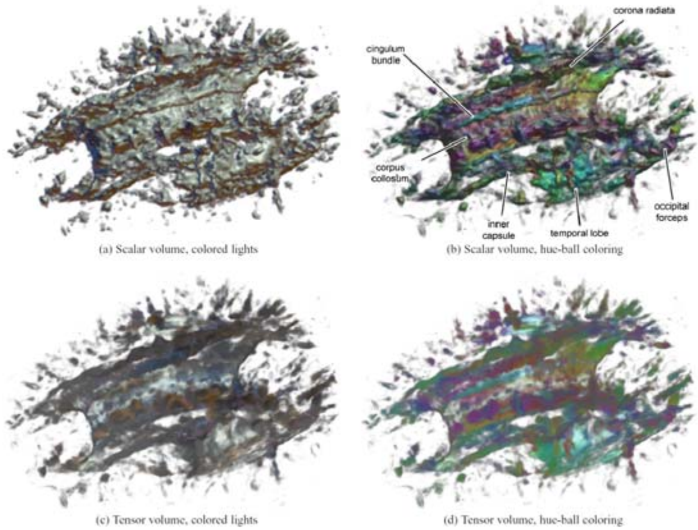
- Lit-tensors
  - Similar to illuminated streamlines
  - Illumination of tensor representations
  - Provide information on direction and curvature
- Cases
  - Linear anisotropy: same as illuminated streamlines
  - Planar anisotropy: surface shading
  - Other cases: smooth interpolation between these two extremes

8.3. Hue-Balls and Lit-Tensors

- Lit-tensors (cont.)
  - Example



8.3. Hue-Balls and Lit-Tensors



(a) Scalar volume, colored lights

(b) Scalar volume, hue-ball coloring

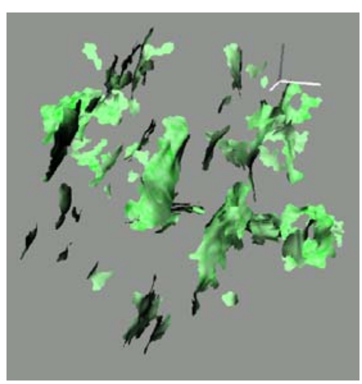
(c) Tensor volume, colored lights

(d) Tensor volume, hue-ball coloring

8.3. Hue-Balls and Lit-Tensors

- Variation: streamtubes and streamsurfaces [Zhang et al. 2000]
  - Streamtubes: linear anisotropic regions
  - Streamsurfaces: planar anisotropic surfaces

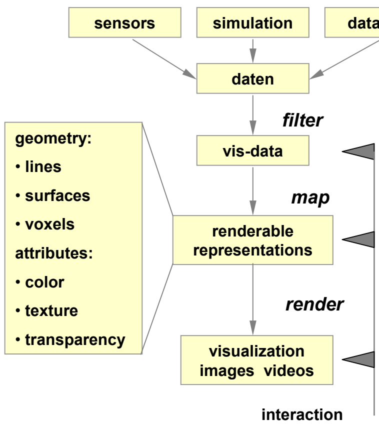
linear



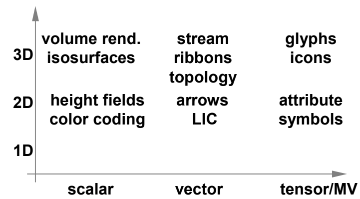
planar

Visualization – pipeline and classification

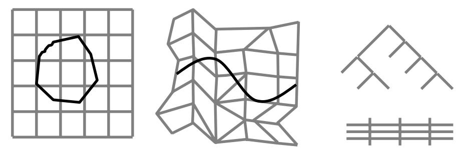
visualization pipeline



mapping – classification



different grid types → different algorithms

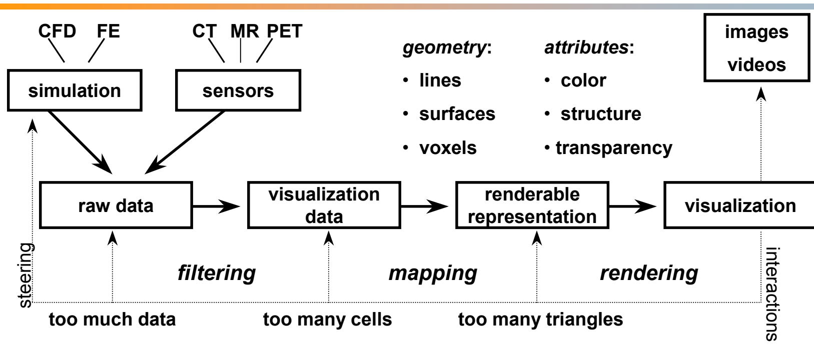


3D scalar fields
cartesian
medical datasets

3D vector fields
un/structured
CFD

trees, graphs, tables,
data bases
InfoVis

Interactive Visualization of Huge Datasets



hierarchical representations

mesh optimization

feature extraction

adaptive algorithms

polygon reduction

progressive techniques

scene graph-optimization

graphics hardware (i.e. textures)

Optimization of all steps of the visualization pipeline

Employ graphics hardware in rendering, mapping, and filtering