Taming Liquids for Rapidly Changing Targets

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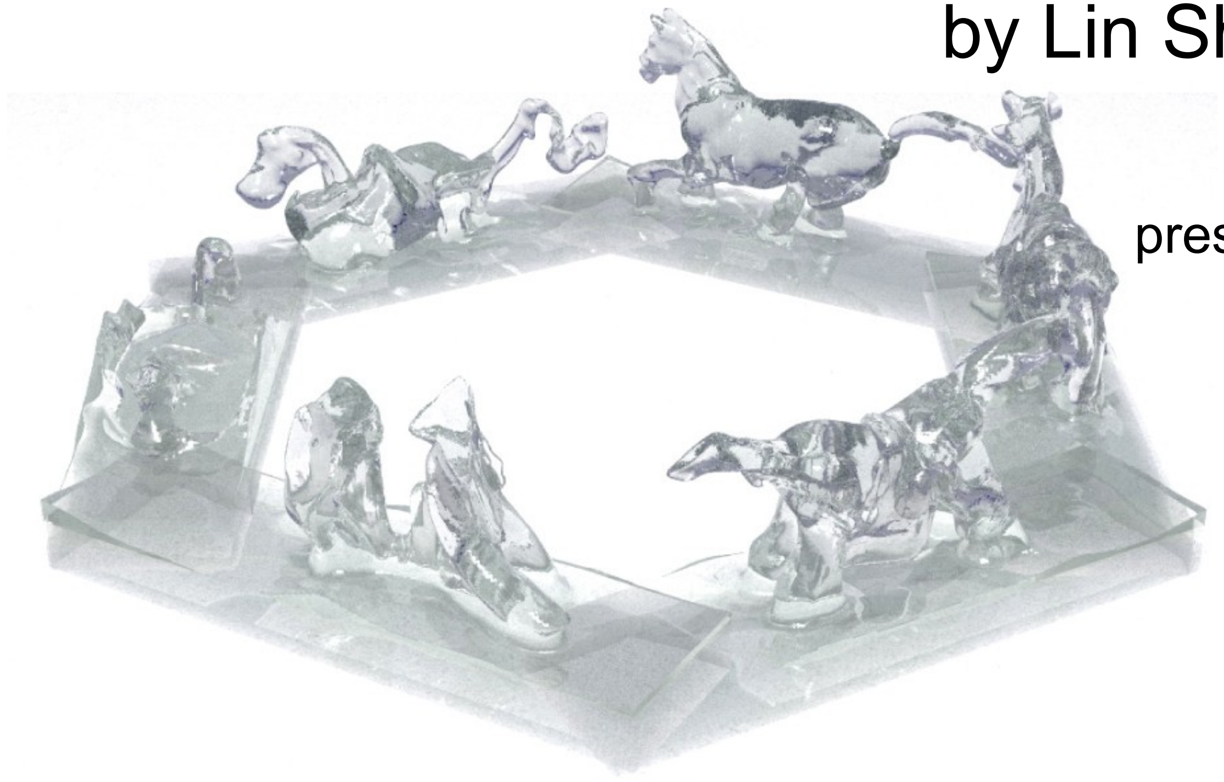
presented by Urban Weber



Outline

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- Feedback control forces
- Adaptive geometric potential
- Results
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- Conclusion

Introduction

- Let the liquid follow a target shape, but preserve it's natural motion
- Challenging problem in fluid control
- Criteria:
  - Control capability
  - Ease to use
  - Fluid-like motion
  - Stability

Workflow

- Animator prepares a continuous sequence of frames („target shape“)
- Three (!) tunable parameters

- Please wait... a few hours

Basic concept

- Apply two external force fields:
  - A feedback force field to compensate discrepancies in both shape and velocity
  - A gradient field of a potential function defined by the target's shape and skeleton
- Solve the Navier-Stokes equations

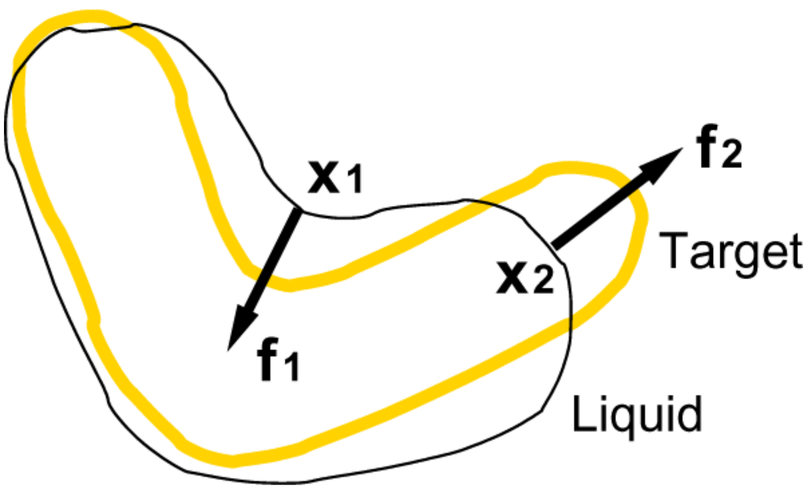
$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} = -(u \cdot \nabla) u - \frac{1}{\rho} \nabla p + \nu \nabla^2 u + f$$

Feedback control forces

- Apply feedback forces for shape and velocity differences
- Complete feedback force:

$$f_{feedback} = f_{shape} + f_{velocity}$$

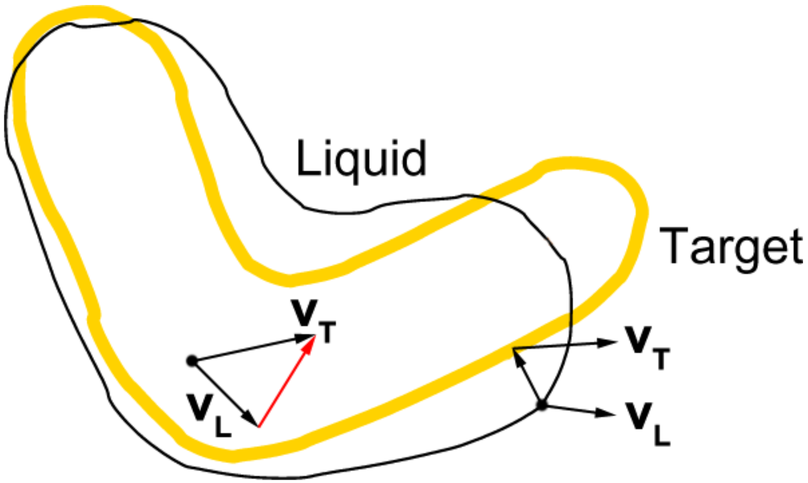


Velocity feedback

- Force field throughout the liquid volume

$$f_{velocity}(x)=\beta(v_T(x)-v_L(x))$$

- Points outside the target shape are projected onto the target



Shape feedback

- Three steps to obtain the force field:
  1. Initialize forces on the boundary
  2. Make sure the flux on the boundary is 0 (to keep the liquid volume-preserving)
  3. Solve a divergence-free force field throughout the liquid (mind the projection step!)

1. Force initialization

- Look at a point x on the liquid boundary:
  - x outside the target shape: $\tilde{f}(x) = -\alpha d_T \frac{\nabla d_T}{\|\nabla d_T\|}$
  - otherwise $\tilde{f}(x) = \alpha d_L \frac{\nabla d_L}{\|\nabla d_L\|}$

$d_T(x,t)$, $d_L(x,t)$: signed distance functions

2. Force optimization

- Total flux must be zero: $\Phi_f = \sum_{i=1}^{m} f_i \cdot n_i = 0 \quad (1)$
- Adjust initial forces by minimizing $\sum_{i=1}^{m} \| f_i - \tilde{f}_i \|^2$ while maintaining (1)
- Solve minimization problem by introducing Lagrange multipliers
- Suppose $\Phi_{\tilde{f}} = \sum_{i=1}^{m} \tilde{f}_i \cdot n_i$ then $f_i = \tilde{f}_i - \frac{\Phi_{\tilde{f}}}{m} n_i$

Shape feedback

3. Solve force field

- Formulate the force field as: $f_{shape} = \nabla H$
- Ensure zero divergence: $\nabla \cdot f_{shape} = \nabla \cdot \nabla H = 0$

- Thus, we get the following Laplace equation with a boundary condition

$$\nabla^2 H = 0, \quad \nabla H|_{\partial \Omega} = f|_{\partial \Omega}$$

which can be efficiently solved by a conjugate gradient method

Adaptive geometric potential

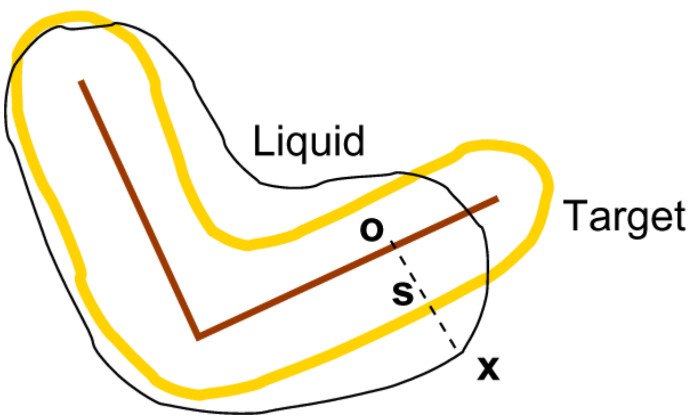
- Why another force field?
- Idea: use the (negative) gradient of a potential field as another control force
- Define the potential as an increasing function of the signed distance:

$$\Phi(x) = C\, sgn(d_T(x)) \left| d_T(x) \right|^{\gamma}$$

- However, thick regions can tolerate more deviation in shape than thin regions

Adaptive geometric potential (2)

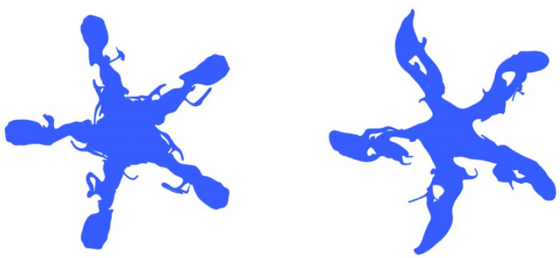
- Therefore, an adaptive signed distance function with respect to the target object's shape AND skeleton is used
- d_0 and d_1 are the skeleton's and the surface's distance values



$$d_T^a(x) = d_0 + (d_1 - d_0)\frac{\|ox\|}{\|os\|}$$

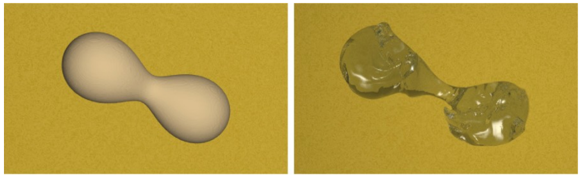
$$d_T^a(x) = -1 + \frac{\|ox\|}{\|os\|}$$

Results



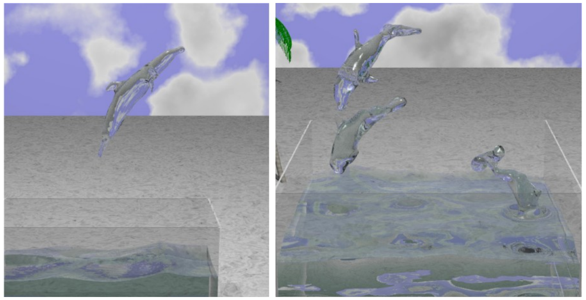
Rotating star shape

(1000^2, 2.5 min/frame)



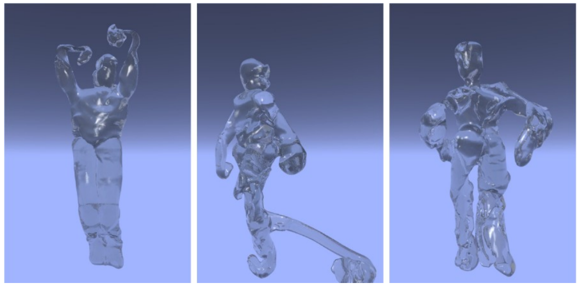
Dumbbell

(300^3, 3.2 min/frame)



Liquid dolphin(s)

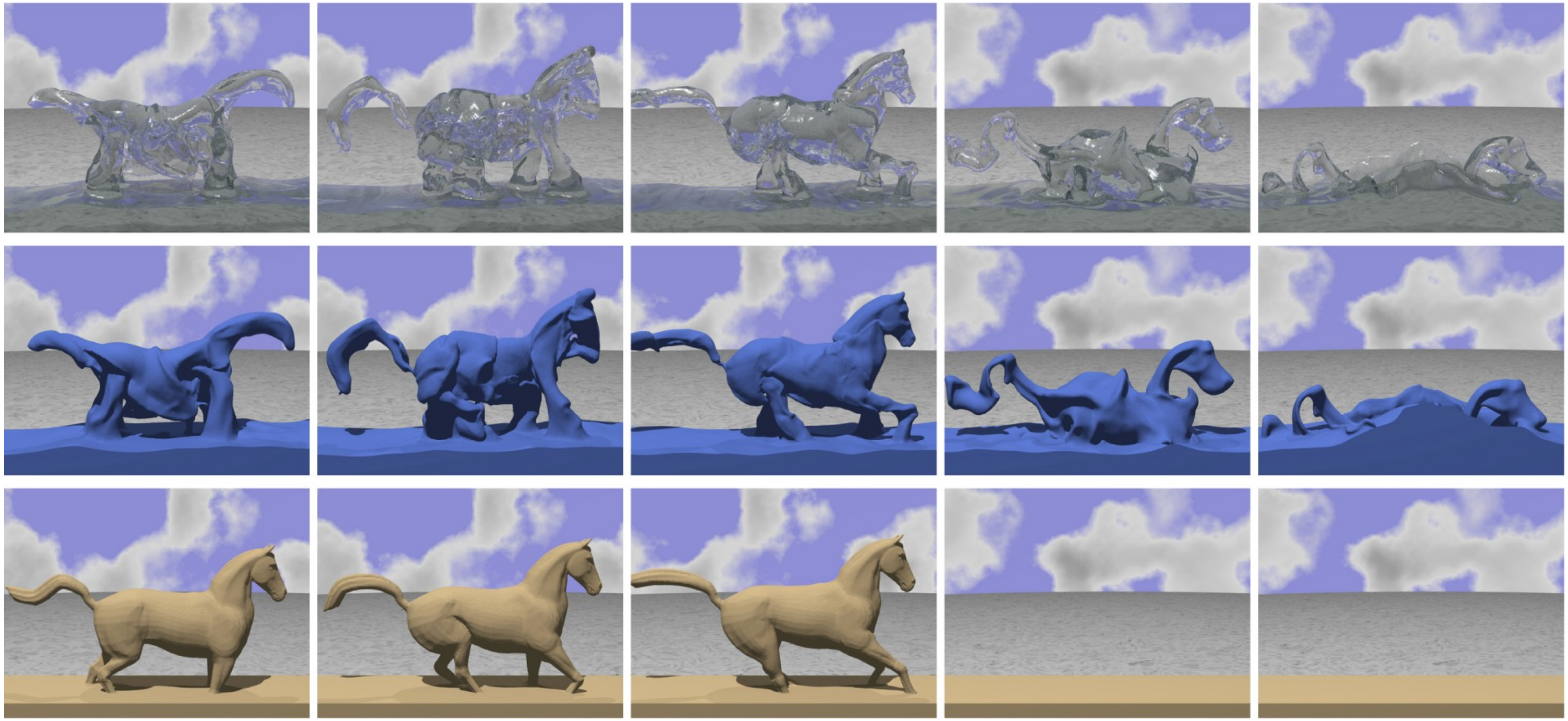
(4.8, 4.2 min/frame)



Dancing sequence

(300^3, 7 min/frame)

Results (2)



Water horse (275x250x75, 4.4 min/frame)

Summary and Analysis

- The input is a continuous animation of a target shape, created from
  - Keyframes
  - Physics-based simulation
  - Motion capture
  - etc.
- The output is a liquid simulation that approximately follows the target

Three types of force fields

- The shape feedback control forces
  - Make the liquid follow the target shape
  - Too strong, resulting in unnatural motion
- The (gradient of the) geometric potential
  - Resembles gravity, appears natural
  - Problems when it's changing quickly from frame to frame (oscillations)
- The velocity feedback forces
  - Heavily reduces oscillations

Summary and Analysis

Performance etc.

- Computational time is almost the same as a regular liquid simulation
- Computing the force fields costs less than 10% of total simulation time

- Grid resolution up to 300^3
- The force fields are relatively insensitive to the grid resolution

Conclusion

- Not only did they achieve their goals, but also managed to do it in a reasonable time.
- No need for drastically changing the parameters for different target shapes.