



Perspective in Art



Giorgio de Chirico: Mystery and Melancholy of a Street

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from: Agrawala, Zorin, Munzner: Artistic Multiprojection Rendering, EGWR 2000

4 Projections



Perspective in Art



Raphael: School of Athens

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from: Agrawala, Zorin, Munzner: Artistic Multiprojection Rendering, EGWR 2000

4 Projections



Perspective in Art



Cezanne: Still Life with Fruit Baskets

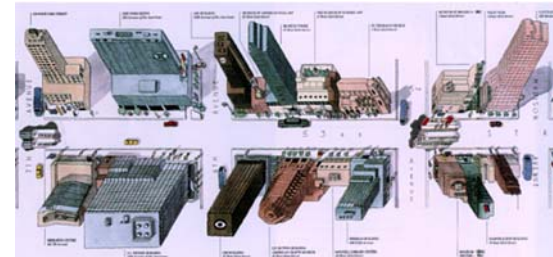
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from: Agrawala, Zorin, Munzner: Artistic Multiprojection Rendering, EGWR 2000

4 Projections



Perspective in Art



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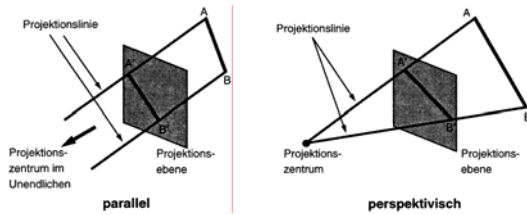
from: Agrawala, Zorin, Munzner: Artistic Multiprojection Rendering, EGWR 2000

4 Projections



Parallel vs. Perspective Projection

- Rigorous mathematical treatment in **Projective Geometry**
- Planar Projections only

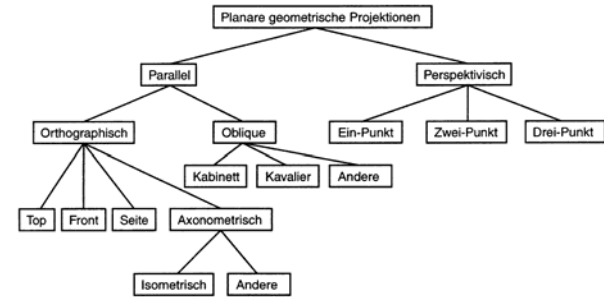


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4. Projections



Classification



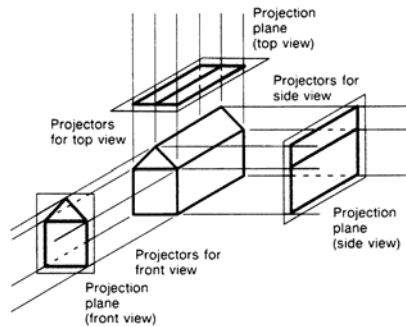
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4. Projections



Orthographic Projection

- Front-, top-, and side views



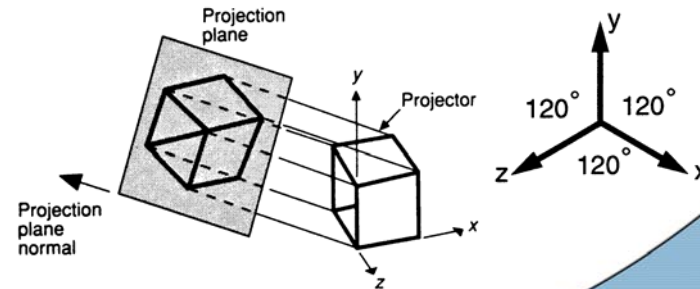
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4. Projections



Isometric Projection

- Projection plane normal equals $(1, 1, 1)$



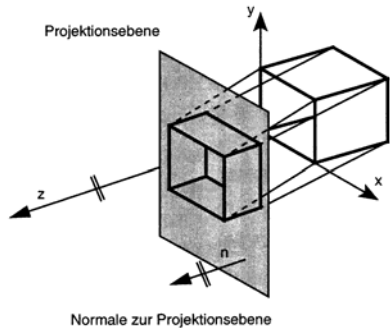
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4. Projections



Oblique Projections

- Normal \neq Direction of Projection

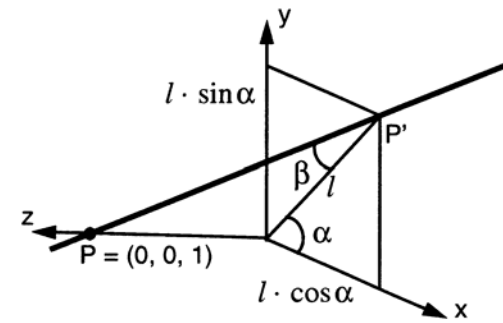


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4 Projections



Direction of Projection



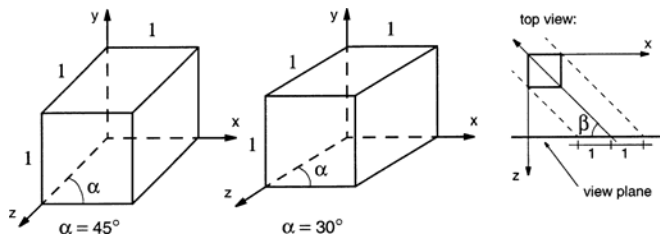
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4 Projections



“Kavalier” Projection

- $\beta = 45^\circ$



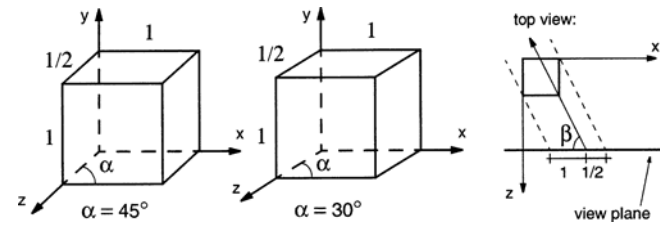
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4 Projections



“Kabinett” Projection

- $\beta = 63.43^\circ$



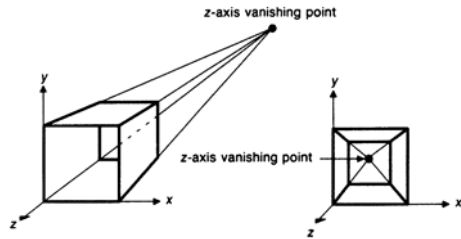
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4 Projections



Perspective Projection

- 1, 2, or 3 vanishing points
- Defined by number of intersections between projection plane and coordinate axes

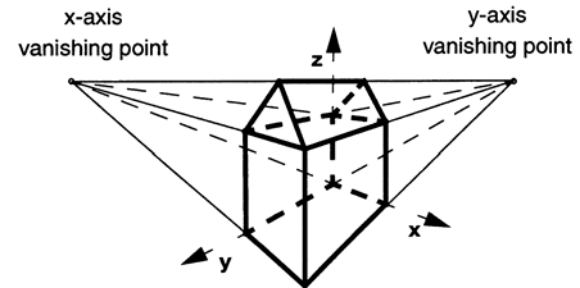


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4 Projections



2 Vanishing Points

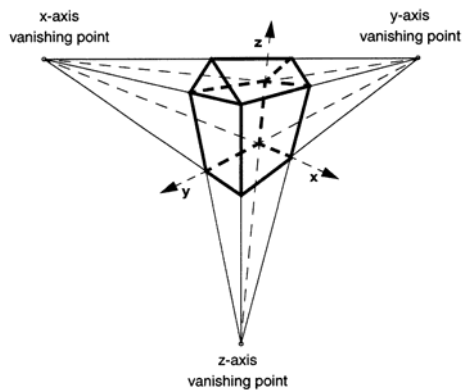


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4 Projections



3 Vanishing Points

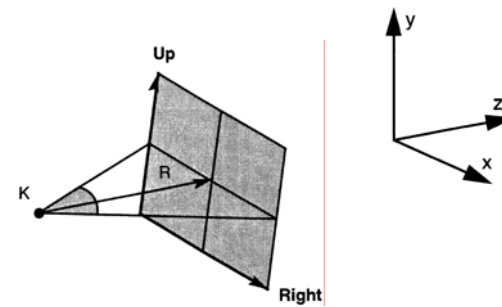


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4 Projections

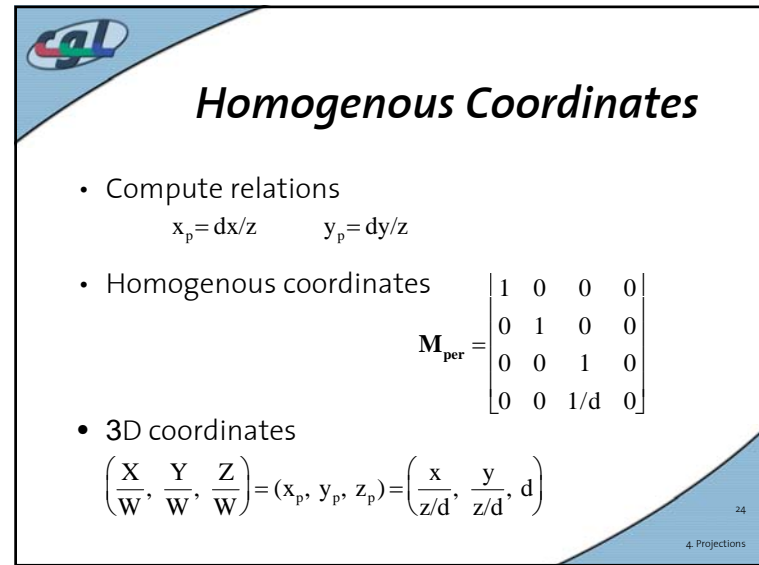
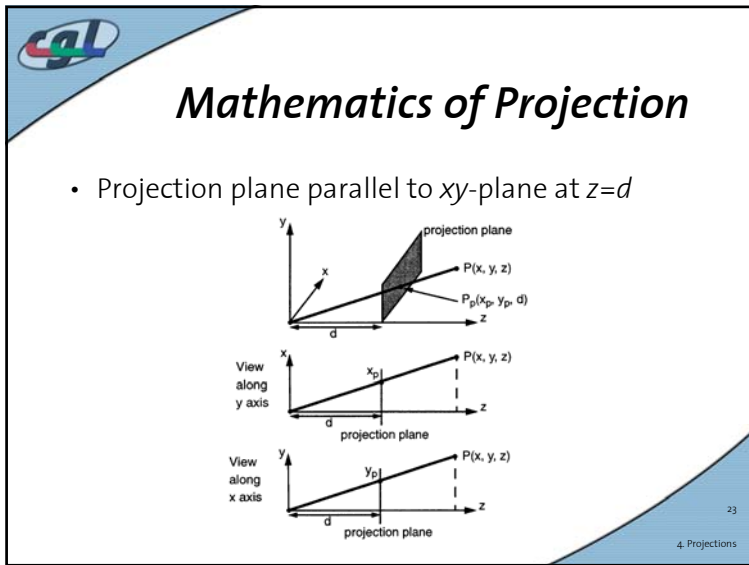
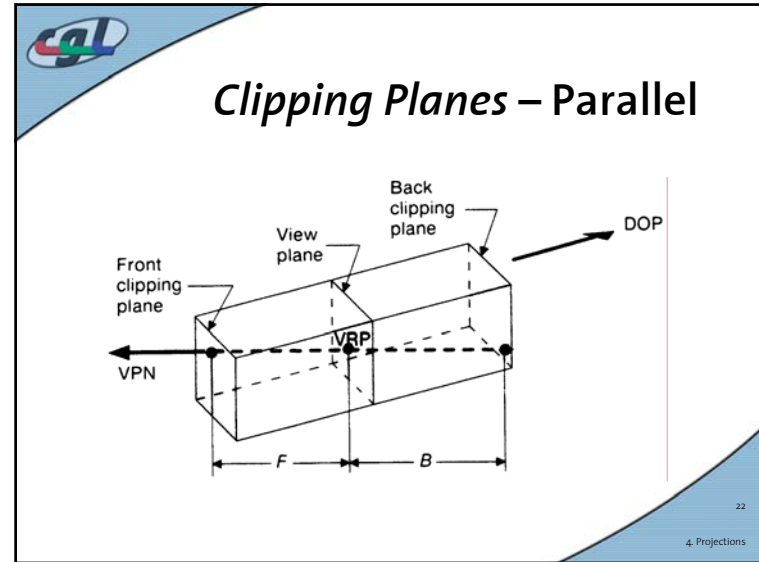
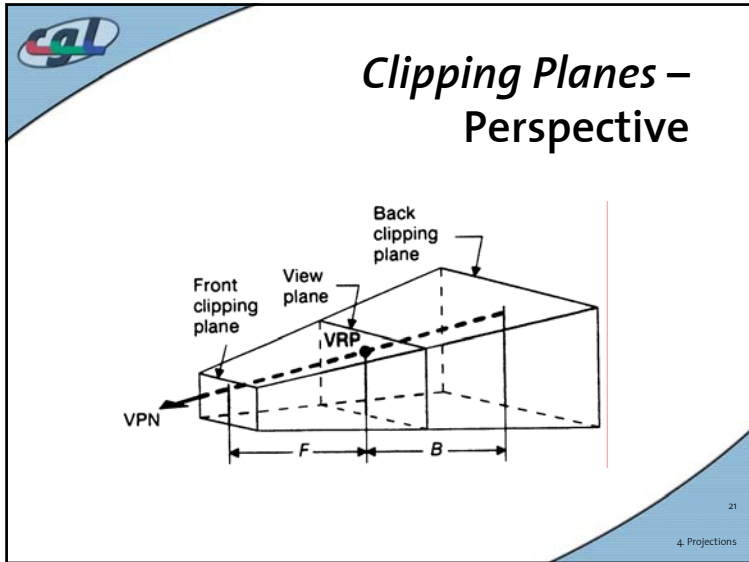



Left-Handed Camera Coordinates



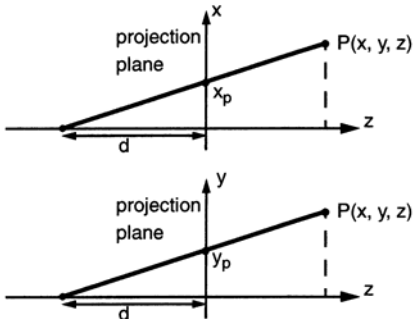
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4 Projections






Z=0 Plane



$$M'_{\text{per}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix}$$

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4 Projections




Parallel Projection

- $d \rightarrow \infty$: yields matrix for parallel projection

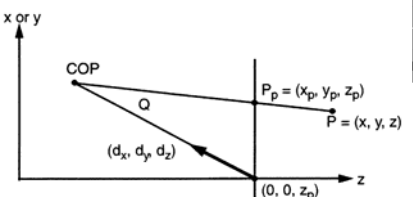
$$M_{\text{ort}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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4 Projections




General Setting

- Arbitrary center of projection (COP)



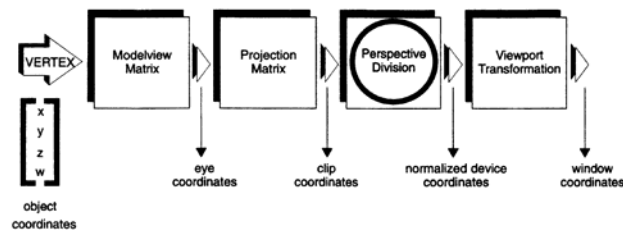
$$M_{\text{general}} = \begin{bmatrix} 1 & 0 & -\frac{d_x}{d_z} & z_p \frac{d_x}{d_z} \\ 0 & 1 & -\frac{d_y}{d_z} & z_p \frac{d_y}{d_z} \\ 0 & 0 & -\frac{z_p}{Q d_z} & \frac{z_p^2}{Q d_z} + z_p \\ 0 & 0 & -\frac{1}{Q d_z} & \frac{z_p}{Q d_z} + 1 \end{bmatrix}$$

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4 Projections



Projections in OpenGL

- Stages of Vertex Transformation

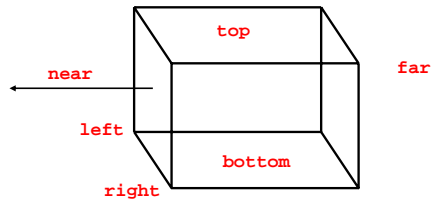


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4 Projections



Projections in OpenGL

- Parallel Projection (*Orthographic Projection*)



```
glOrtho(left, right, bottom, top, near, far);
```

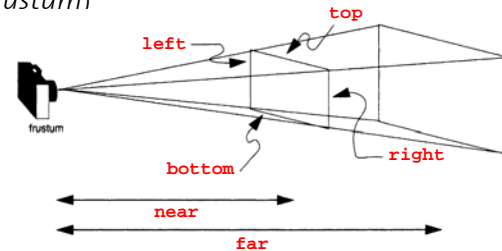
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4 Projections



Projections in OpenGL

- Perspective Projection (*Definition of a Frustum*)



```
glFrustum(left, right, bottom, top, near, far);
```

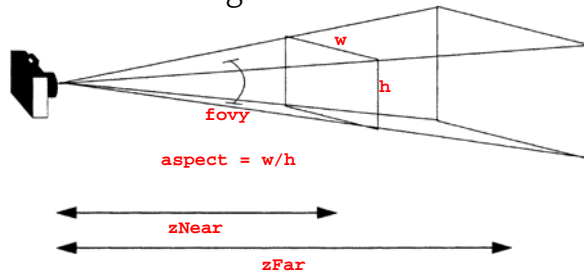
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4 Projections



OpenGL-Utility Functions for Defining Projections

- Camera Analogon



```
gluPerspective(fovy, aspect, zNear, zFar);
```

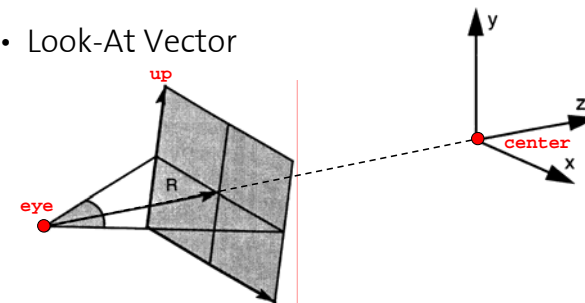
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4 Projections



OpenGL-Utility Functions

- Look-At Vector



```
gluLookAt(eyeX, eyeY, eyeZ,
centerX, centerY, centerZ,
upX, upY, upZ);
```

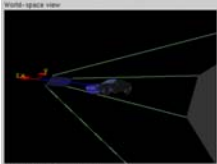
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4 Projections

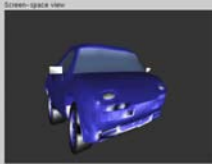


The OpenGL Camera

World-space view



Screen-space view



Command manipulation window

```
glTranslatef( 0.00 , 0.00 , 0.00 );  
glRotatef( 43.0 , 0.34 , 1.00 , 0.00 );  
glScalef( 1.99 , 2.01 , 0.64 );  
glBegin( ... );  
...  
Click on the arguments and move the mouse to modify values.
```

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