

Concept of Texture Mapping

- Find mappings between different coordinate systems
- Invert transformation from texture coordinates to image pixel

Texture coordinates u, v

Surface of object $w(u, v) = (x, y, z)$

Pixel coordinates x, y

Four corners of pixel on screen

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Parameterization

- Find a one-to-one mapping between given surface and 2D parameter domain

\mathbb{R}^3 S \mathbb{R}^2 Ω

u v

$U(x, y, z)$ $X(u, v)$

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Parameterization

- Fundamental concept in graphics
- Many different applications
 - Morphing

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Parameterization

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Some History


- Cartography




orthographic



stereographic
↑
preserves angles
= conformal



Mercator



Lambert
↑
preserves area
= equiareal

Floater, Hormann: *Surface Parameterization: A Tutorial and Survey*, Advances in Multiresolution for Geometric Modeling, 2005

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Analytical 3D Surfaces

- Key to texture mapping: **Parameterization**


$$\begin{bmatrix} s \\ t \end{bmatrix} \rightarrow \begin{bmatrix} x(s,t) \\ y(s,t) \\ z(s,t) \end{bmatrix} \quad \text{sphere: } \begin{bmatrix} \theta \\ \phi \end{bmatrix} \rightarrow \begin{bmatrix} \sin \theta \sin \phi \\ \cos \phi \\ \cos \theta \sin \phi \end{bmatrix}$$

- Map parameters to texture coordinates

$$\begin{bmatrix} s \\ t \end{bmatrix} \rightarrow \begin{bmatrix} u(s,t) \\ v(s,t) \end{bmatrix} \quad \text{inverse: } \begin{bmatrix} u \\ v \end{bmatrix} \rightarrow \begin{bmatrix} s(u,v) \\ t(u,v) \end{bmatrix}$$

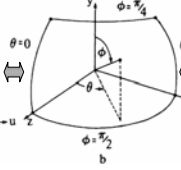
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Mapping a texture onto a sphere




a

\Leftrightarrow



b

\Leftrightarrow



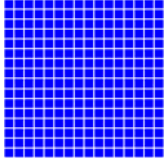

c

- Use linear map

$$\begin{bmatrix} \theta \\ \phi \end{bmatrix} = \begin{bmatrix} Au + B \\ Cv + D \end{bmatrix} \Rightarrow \begin{bmatrix} \theta \\ \phi \end{bmatrix} = \begin{bmatrix} \pi/2 \cdot u \\ -\pi/4 \cdot v + \pi/2 \end{bmatrix}$$

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Example

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Desirable Properties



- Low distortion
- Bijjective mapping
- Efficiently computable

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Additional Issues

- Finding cuts
- Texture Atlases

Levy, Petitjean, Ray, Maillot: *Least Squares Conformal Maps for Automatic Texture Atlas Generation*, SIGGRAPH, 2002

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Additional Issues

- Constraint Texture Mapping

→ Demo

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Texture Map

- Texture map corresponds to parameterization

Tim Weyrich et al.

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Parametrized Triangle Mesh

OBJ Files:

```

v 0.131171 -0.113469 0.178314
v 0.130945 -0.114951 0.182474
v 0.130916 -0.115792 0.185402
...
vt 0.538446 0.4275
vt 0.550132 0.41427
vt 0.546491 0.427631
...
vn 0.609697 0.486474 0.625789
vn 0.799934 0.334347 0.498315
vn 0.942394 0.131824 0.307435
...
f 22/209/22 220/210/220 221/211/221
f 21/213/21 219/214/219 220/210/229
f 253/203/253 219/214/219 21/213/21
...
  
```

Vertex positions

Texture coordinates

Normals

Faces (triangles)
coordNr / texNr / normalNr

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Rasterization

- From texture coordinates of **vertices** to texture coordinates of **pixels**
- Linear interpolation in screen-space (as in Gouraud shading):

Images by Fredo Durand

texture source what we get what we want

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Perspective Interpolation

- Linear variation in world coordinates yields non-linear variation in screen coordinates:

projection plane

- Perspective interpolation implemented in today's graphics cards

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Texture Filtering

- (u, v) are real pixel coordinates, e.g. $(6.4, 3.7)$:



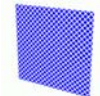
nearest bi-linear


$color = map[6,4]$
 $color = \bar{s} \cdot \bar{t} \cdot map[6,3] + s \cdot \bar{t} \cdot map[7,3] + \bar{s} \cdot t \cdot map[6,4] + s \cdot t \cdot map[7,4]$

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Texture Filtering


+

=





nearest



bi-linear

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Texture Mapping in OpenGL

```

loadImage(stexture_data);
glGenTextures(1, &texId);
glBindTexture(GL_TEXTURE_2D, texId);
glTexImage2D(GL_TEXTURE_2D, 0, GL_RGB,
             w, h, 0, GL_RGB, GL_UNSIGNED_BYTE,
             texture_data);
-
glBindTexture(GL_TEXTURE_2D, texId);
glBegin(GL_TRIANGLES);
    glTexCoord2f(u0, v0); glVertex(x0, y0, z0);
    glTexCoord2f(u1, v1); glVertex(x1, y1, z1);
    glTexCoord2f(u2, v2); glVertex(x2, y2, z2);
glEnd();
    
```

$w = 2^n, h = 2^m$

$u, v \in [0 \dots 1]$

→ Tutor

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