ETH

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Visual Computing

4) Convolution and Fourier Transform (from Mod. & Sim. exam 05/06)

Consider the one-dimensional box function f

$$f(x) := \begin{cases} 1 & \text{if } |x| \leq \frac{1}{2} \\ 0 & \text{otherwise} \end{cases}$$

a) Calculate the Fourier transform of the function $f(x)$.

b) Assume the filter f is applied to a signal $s(x)$: $[f * s](x)$. Which frequencies in the spectrum of s will be lost? Which part of the spectrum will be damped the most: low, medium or high frequency bands?

c) Iterative convolution (central limit theorem):

- Calculate the Fourier transform of the function $g_2(x) := [f * f](x)$.
- Calculate the Fourier transform of the function g_n resulting from convolving n versions of f,
  $g_n(x) := [\underbrace{f * \cdots * f}_{n \text{ times}}](x)$.
- Which filter function is obtained for $n \to \infty$? Draw a qualitative sketch of f, g_2, and g_3 and observe the convergence.

d) Assume the filter $g_{\lim} := \lim_{n \to \infty} g_n$ is applied to a signal $s(x)$: $[g_{\lim} * s](x)$. Which frequencies in the spectrum of s will be damped the most: low, medium, or high ones? Which frequencies will be erased completely?