A Probe for Local Flow Field Visualization

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Abstract

A probe for the interactive visualization of flow fields is presented. The probe can be used to visualize many characteristics of the flow in detail for a small region in the data set. The velocity and the local change of velocity (the velocity gradient tensor) are visualized by a set of geometric primitives. To this end, the velocity gradient tensor is transformed to a local coordinate frame, and decomposed into components parallel with and perpendicular to the flow. These components are visualized as geometric objects with an intuitively meaningful interpretation. An implementation is presented which shows that this probe is a useful tool for flow visualization.

1 Introduction

Flow visualization is an important topic for data visualization research. Most of the recently published methods are concerned with global visualization of the flow. Volume rendering [8], flow topology [6] etc. show structures in flows, but do not allow a detailed look at a particular point in the flow field. The probe described here is intended as a complement to global visualization methods. The aim is to provide detailed information at a point by visual means and powerful user interaction.

A simple arrow only shows the magnitude and direction of the flow velocity at a point, but often not just velocity but changes in velocity are of interest. These velocity changes can be described by a second order real tensor. Therefore visualization of the local flow is in essence representation of a vector (velocity) and a tensor (velocity changes) in a point. To this end, meaningful values are extracted from the tensor which are represented by geometric objects.

In section 2 a brief introduction to tensors and their visualization is given. In section 3 a new method for the decomposition of a second order real tensor is proposed. This decomposition is useful and intuitively clear in flow fields. In section 4 the subsequent mapping on geometric objects is presented. In section 5 an implementation is described. Finally, in section 6 the results are discussed.

2 Tensor visualization

In a vector field the first order approximation of $\mathbf{u}(\mathbf{x})$ near a point $\mathbf{x}_0$ is:

$$\mathbf{u}(\mathbf{x}) = \mathbf{u}(\mathbf{x}_0) + J(\mathbf{x} - \mathbf{x}_0) \ . \tag{1}$$

In three dimensions the velocity gradient or Jacobian matrix [1, 7] of a velocity field $\mathbf{u}(\mathbf{x}) = (u, v, w)$ is given by:

$$J = \nabla \mathbf{u} = \begin{pmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{pmatrix} . \tag{2}$$

A subscript denotes a partial derivative, so u_x means $\partial u / \partial x$. To show local flow characteristics in a point a vector $\mathbf{u}$ and a tensor J must be visualized.

2.1 Decomposition

Decomposition is an important technique to achieve insight in a tensor. An often used decomposition is to split tensors in a symmetrical and an anti-symmetrical part [1]:

$$J \ = \ J^{(s)} + J^{(a)} \ , \tag{3}$$

where

$$\begin{aligned} J^{(s)} &= (J + J^T)/2 \quad \text{and} \\ J^{(a)} &= (J - J^T)/2 \ . \end{aligned}$$

J^T is the transpose of J. The anti-symmetrical part has three independent components which determine the rotation of the velocity. The symmetrical part of the tensor is called the stress-strain tensor. If the directions of the orthogonal axes of reference are chosen such that the non-diagonal elements of the stress-strain tensor are zero, as is always possible, then the tensor represents a pure stretching motion. The axes of this frame are called the principal directions of the tensor. The axis along which the largest stretching occurs is called the major principal direction.

The values of a real tensor can be interpreted in terms of the distortion of an infinitesimal fluid element around a point where the tensor is defined. In this way the decomposition of the tensor can be interpreted as imposing certain restrictions on the possible distortions of the fluid element. The tensor can be viewed as a superposition of those distortions. Decomposition in a symmetrical and anti-symmetrical part is the result of using a rigid object that can only rotate and a second object that can change shape but not rotate. This is not the only way the tensor can be decomposed. Other ways of decomposition result in other restrictions on the movement of the fluid element. For example, the symmetric part can be further decomposed into an isotropic expansion and two shearing motions[1].

2.2 Previous work

Several researchers have devised graphical representations of tensors. Haber [5] represented a symmetric stress tensor by an object consisting of a shaft and a disc. The direction of the shaft is the major principal direction of the tensor. The axes of the elliptical disc correspond to the other two principal directions. The length of the shaft and the smallest and largest radius of the ellipse are proportional to the magnitudes of the eigenvalues of the tensor.

The Stream Polygon by Schroeder et al. [11] is used to visualize local deformation. It is a regular n-sided polygon perpendicular to the velocity vector. Data is mapped onto attributes of the polygon, such as the radius and the relative length of the edges. Deformation can be visualized by deforming the polygon accordingly. By sweeping the polygon along a streamline a stream tube results. Scalar values can be mapped on the surface of this tube by using color.

Delmarcelle and Hesselink [4] show that a tensor field can be decomposed in a symmetric-tensor field and a vector field. To visualize the global structure of this symmetric-tensor field they use hyperstreamlines. A hyperstreamline is an ellipse swept along the line everywhere tangent to the major principal direction of the tensor. The magnitude and direction of the other two principal directions at each point are reflected in the size of the swept ellipse. The resulting surface can be colored according to the magnitude of the principle direction.

The presented methods show a tensor field separated from the velocity field, and are therefore not very useful to visualize local flow. Further, they only show the symmetric part of the tensor while the velocity gradient is in general an asymmetrical tensor.

3 Local flow

To visualize local flow via a probe, meaningful quantities must be derived from the raw data. In this section the mathematical basis for the probe is presented. The construction of a local coordinate frame is described, and next the tensor is transformed to this local coordinate frame and decomposed into primitives.

Because flow data usually are velocity vectors defined at grid nodes, the velocities are known only at discrete locations and must be interpolated in between. For this we use trilinear interpolation. The gradients are calculated by centered differences [10]. So far we limited ourselves to rectilinear grids. Extension to curvilinear grids and irregular meshes is possible, but requires a more complex numerical evaluation of the Jacobian matrix.

3.1 Local coordinate frame

The definition of a tensor implies its invariance for coordinate changes. In fluid flow, however, a distinction can be made between velocity changes *parallel with* the flow and *perpendicular to* the flow. This can be used to decompose the tensor in components parallel with the flow and components perpendicular to the flow. Instead of a fluid element, a "fluid line" and a "fluid plane" are used to map the distortion. Together these two elements can represent the same information as the fluid element mentioned before. Therefore, the velocity gradient tensor is transformed to a local coordinate frame (fig. 1). The origin of this frame is the point where the tensor is calculated. For the frame we use a Frenet frame [3, 2] with the x-axis parallel with the velocity vector and the y-axis parallel with the curvature vector.

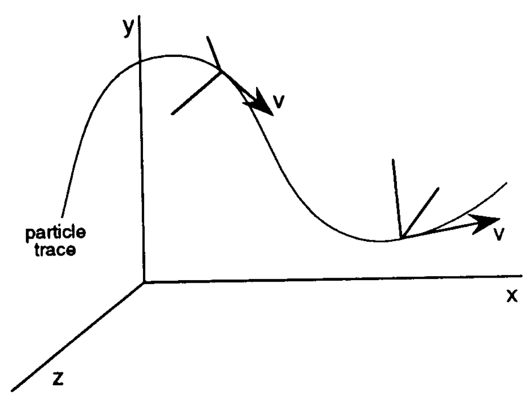

Figure 1: Two local coordinate frames

In order to define a Frenet coordinate frame, the curvature of a streamline through the origin has to be calculated.

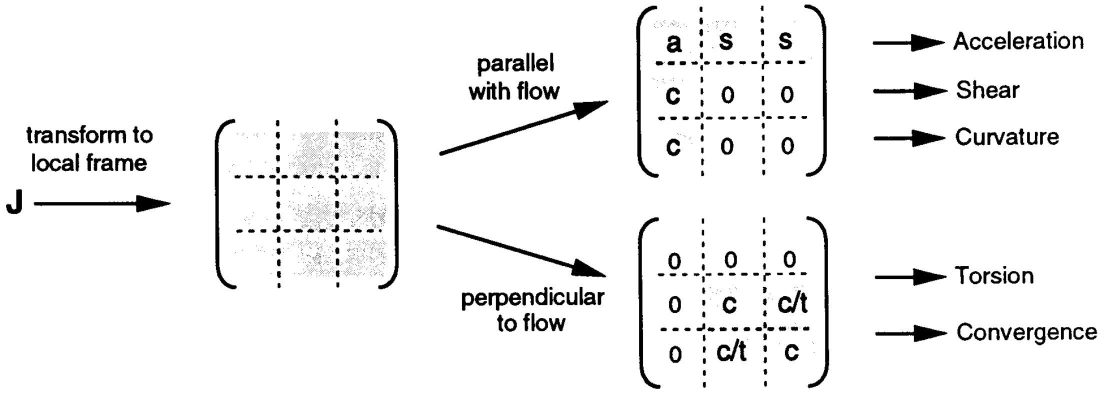

Figure 2: Decomposition of the probe

The curvature vector $\mathbf{c}$ of a curve $\mathbf{p}(s)$ through the origin is given by [3]:

$$\mathbf{c} = \frac{d^2\mathbf{p}}{ds^2}\,, \tag{4}$$

where s is the parameter of path length. Using the first order approximation of the velocity of an imaginary particle that moves along the streamline:

$$\mathbf{u}(t) = \mathbf{u}_0 + J\mathbf{u}_0 t \tag{5}$$

and using the fact that

$$\frac{d\mathbf{p}}{ds} = \frac{\mathbf{u}(t)}{|\mathbf{u}(t)|}\,, \tag{6}$$

it can be shown that the curvature vector at a point in the flow is given by:

$$\mathbf{c} = \frac{J\mathbf{u}(\mathbf{u}\cdot\mathbf{u}) - \mathbf{u}(\mathbf{u}\cdot J\mathbf{u})}{|\mathbf{u}|^3}\,. \tag{7}$$

With this vector and the velocity vector the Frenet frame can be constructed. The base of the frame consists of the normalized velocity vector, the normalized curvature vector and the cross product of those two vectors:

$$\left(\frac{\mathbf{u}}{|\mathbf{u}|}, \frac{\mathbf{c}}{|\mathbf{c}|}, \frac{\mathbf{u}\times\mathbf{c}}{|\mathbf{u}\times\mathbf{c}|}\right) \tag{8}$$

If the curvature is zero the Frenet frame is not defined. In this case any orthonormal frame with one axis aligned with the velocity can be used.

3.2 Decomposition and mapping

In order to construct the probe, the tensor is decomposed into components in the direction of and perpendicular to the velocity (fig. 2). The components in the velocity direction are both the derivatives with respect to x (u_x, v_x and w_x) and the velocity component in the $\mathbf{u}$ direction (u_x, u_y and u_z).

The components in the perpendicular direction (v_y, v_z, w_y and w_z) form the 2×2 lower-right submatrix of the Jacobian matrix. This last matrix is split in a symmetric and anti-symmetric part. Just like in the three-dimensional case a rotation and a stress-strain-tensor result. These relate to torsion (rotation around the velocity vector) and convergence of the flow.

The velocity parallel components are split as follows. The left column contains the velocity change with respect to x (u_x, v_x, w_x). The top row contains the change of the velocity in the x-direction (u_x, u_y, u_z). The top left element of the matrix (u_x) belongs to both the left column and the top row and gives the acceleration. The remaining elements in the top row (u_y and u_z) give the shear in the flow, and the remaining elements in the first column (v_x and w_x) give the curvature of the streamline.

4 The probe

The previous section covered the mathematics of the tensor decomposition used for the probe. This section covers the mapping of the derived quantities to geometric primitives. The tensor is decomposed in five components: curvature, shear, acceleration, torsion and convergence. The acceleration and curvature are derived from the change in the velocity of a particle in the flow. The other three primitives show the velocity changes as seen from a ring or a disc perpendicular to the velocity. Fig. 3 shows a graphical representation of the probe.

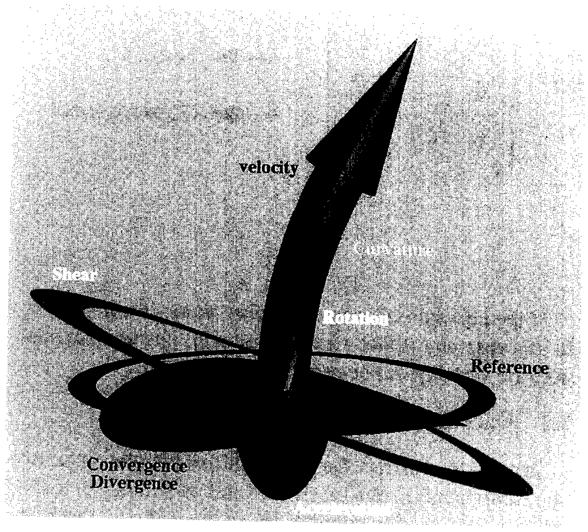

Figure 3: Components of the probe

4.1 Requirements

If decomposition is used then care has to be taken that the resulting components are independent. This is important mathematically as well as visually. Mathematically an orthogonal decomposition is desirable so that the different components can each change their value without affecting the other components. Also, the visualization of one component should not interfere with the visualization of the other. A primitive should for example not become arbitrarily large if the corresponding value in the tensor increases. Another requirement is that it must be clear if the value of a certain variable is zero.

4.2 Velocity

The velocity is represented by an arrow in the form of a cylinder and a cone. Its direction can be derived from the shading of the arrow. The length of the shaft of the arrow represents the magnitude of the velocity, its direction represents the direction of the velocity. In the local coordinate frame, velocity has only a component in the x-direction, therefore the magnitude velocity can be simply referred to as u.

The length of the velocity arrow is determined by the time scale Δt used. This also affects primitives related to the velocity (curvature, shear and acceleration). The length of the shaft is given by:

$$l = u\Delta t\,. \tag{9}$$

4.3 Curvature

The curvature of the streamline is given by equation (7). In the local frame the center of the osculating circle of the streamline is given by:

$$\mathbf{q} = \begin{pmatrix} 0 \\ u/v_x \\ 0 \end{pmatrix}\,. \tag{10}$$

Note that the first and third components of the vector are always zero because a Frenet frame is used.

The curvature is visualized by the velocity arrow of the probe (fig. 4): the velocity arrow is an arc of the osculating circle. The arrow thus shows a first order approximation of a streamline through the origin of the frame. The angle β over which the arc extends follows from:

$$\beta = \Delta t u/|\mathbf{q}| = \Delta t v_x\,. \tag{11}$$

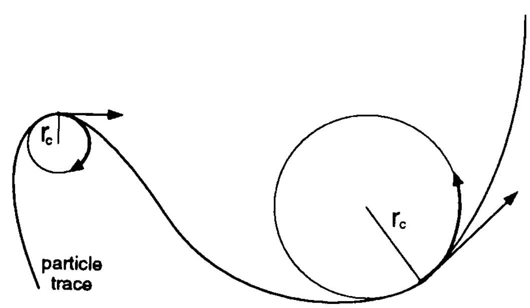

Figure 4: Visualization of curvature

4.4 Acceleration

The value of u_x determines the acceleration of a particle released at the origin of the reference frame. This acceleration is visualized by a "membrane" perpendicular to the flow. The value of the acceleration is mapped to the displacement of the center of the membrane. Zero acceleration

(See color plates, p. CP-5.)

results in a flat membrane. If the acceleration is positive, a half ellipsoid in the direction of the velocity is shown, if it is negative (deceleration), the center of the membrane is stretched in the opposite direction. Thus, it shows the difference in velocity between the current position and at end of the velocity arrow. The displacement a of the center of the membrane is given by:

$$a = u_x \Delta t^2 u \,. \tag{12}$$

The square in the timestep Δt is caused by the fact that both the length of the arrow and the duration of the velocity change has to be taken in account.

4.5 Torsion

The torsion r around the velocity axis is the component of the rotation of the velocity gradient tensor in the direction of the x-axis:

$$r = w_y - v_z \,. \tag{13}$$

The torsion is visualized as candy stripes on the surface of the velocity arrow. The total torsion angle α around the axis is given by:

$$\alpha = r \Delta t \,. \tag{14}$$

Torsion is closely related to helicity of the flow, helicity is the product of torsion and velocity: $\mathbf{u} \cdot \nabla \times \mathbf{u}$.

4.6 Shear

The shear in the direction of the flow can be interpreted as the change of orientation of the plane perpendicular to the flow over time (fig. 5). This shear-plane is visualized by a ring resulting from clipping the plane to two spheres of different size. The equation of the plane of the ring is:

$$x = \Delta t (u_y y + u_z z) \,. \tag{15}$$

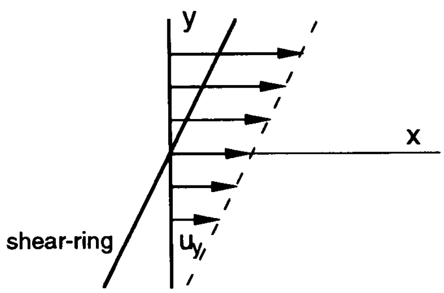

Figure 5: Visualization of shear

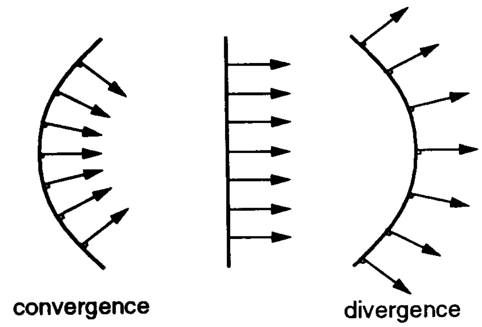

Figure 6: Visualization of convergence

4.7 Convergence

Velocity changes in the plane perpendicular to the flow are related to convergence and divergence of the flow. They affect the size and orientation of a circle released perpendicular to the flow. For visualization a surface that is everywhere perpendicular to the flow is used. This surface can be interpreted as a "lens" that focuses the flow (fig. 6). Here we use the osculating paraboloid [3] to this surface to represent its curvature, just as we used an osculating circle to represent the curvature of a streamline. The distance $\delta(\eta, \xi)$ of an osculating paraboloid to the tangent plane with unit normal $\mathbf{N}$ is given by:

$$\delta(\eta, \xi) = \frac{1}{2}(L d\eta^2 + 2M d\eta d\xi + N d\xi^2) \,, \tag{16}$$

where

$$\begin{array}{rcll} L & = & -(\mathbf{x}_\eta \cdot \mathbf{N}_\eta) & , \\ M & = & -\frac{1}{2}[(\mathbf{x}_\xi \cdot \mathbf{N}_\eta) + (\mathbf{x}_\eta \cdot \mathbf{N}_\xi)] & \text{and} \\ N & = & -(\mathbf{x}_\xi \cdot \mathbf{N}_\xi) & . \end{array}$$

η and ξ are the parameters of the normal plane. For the osculating paraboloid to the surface perpendicular to velocity $[(\mathbf{x}_\eta \cdot \mathbf{u}) = (\mathbf{x}_\xi \cdot \mathbf{u}) = 0]$, L is given by:

$$L = -\mathbf{x}_\eta \cdot \mathbf{N}_\eta = -\mathbf{x}_\eta \cdot \frac{\mathbf{u}_\eta}{|\mathbf{u}|} \,. \tag{17}$$

M and N can be determined similarly. If y and z of the reference frame are chosen as the surface parameters η and ξ, then the equation for the surface perpendicular to the velocity reduces to:

$$x = -\frac{v_y}{u} y^2 - \frac{(v_z + w_y)}{u} yz - \frac{w_z}{u} z^2 \,. \tag{18}$$

This equation is used to construct the surface. Rendering the surface as a disc with a fixed radius as domain poses problems when the differentials are large: the edges will be far from the probe. To prevent this, the surface is clipped against a sphere around the origin.

5 Implementation

To test the concept of the probe an interactive application was written. The application was implemented on a SGI workstation using the C programming language. The user interface was built using 'Forms' [9], a public domain user interface toolkit. For testing purposes simple artificial data sets [12], and a data set generated by Robert Wilhelmson of NCSA were used. This set is the result of a numerical simulation of a tornado generated assuming symmetry in the azimuthal direction.

5.1 User interface

The application provides the user with two views on the flow area (fig. 7). The first is a global view, which shows the position of the probe in the flow area. The second view gives an enlarged view of the probe. The user can interact with the probe in both views. In the global view the user can position the probe at a certain location. Its position in the flow area is indicated by two lines connected to the bounds of the area. In the enlarged view the user can choose a point of view to the probe.

If the user is not interested in certain aspects of the flow, the corresponding primitives can be turned off. Copies of the probe or components of it can be left behind at interesting positions (fig. 8).

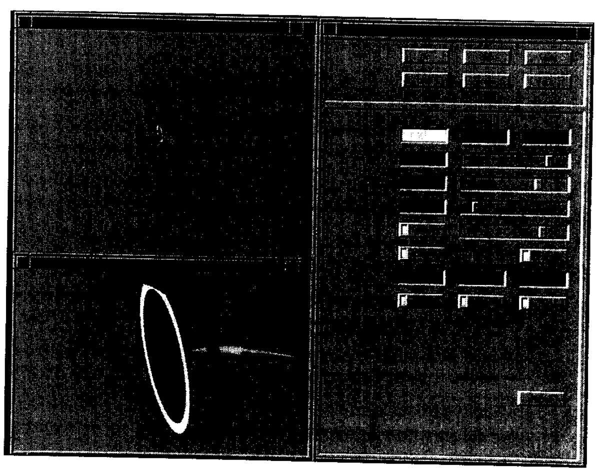

Figure 7: User interface of the application

In the program, animations of a probe moving along a stream line can be generated in real time. The user can select a position in the flow and start the animation of the probe. The path of the probe is shown during the animation. With these additional tools a more global impression of the flow can be achieved.

Figure 8 is an example of how the probe can be applied

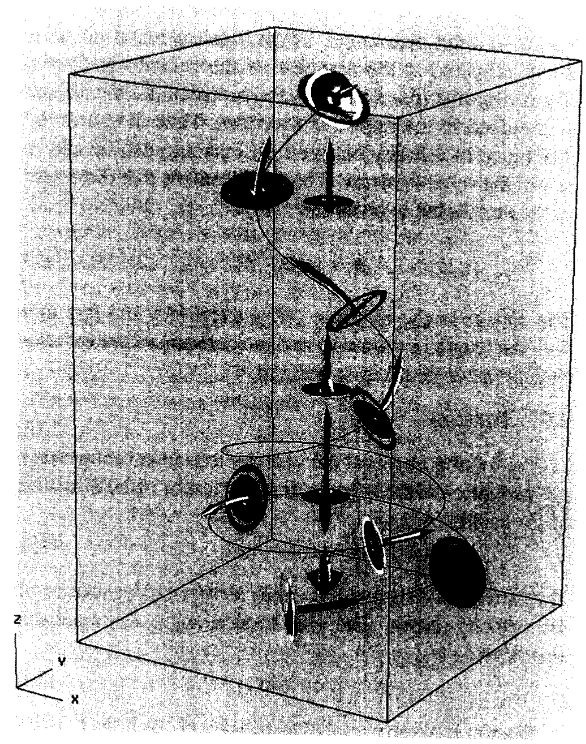

Figure 8: Interactively built image of the simple tornado set (data: Robert Wilhelmson, NCSA)

to a data set. The storm data set is portrayed by several probes. These probes are interactively placed by the user to show interesting locations in the flow. The changes of velocity in the vertical direction along the eye of the tornado are clearly visible. Furthermore, a streamline with several probes along it is shown.

5.2 Display of other data

To show other data at the location of the probe, such as temperature or pressure, a "valuebox" is used. The valuebox is a small display with a constant position relative to the probe on the screen. If the probe is moving or moved the box moves along with it on the screen. The value of the property is updated according to the current position of the probe. This valuebox can also be used for the values already visualized by the shape of the probe, so the exact value of the property at a certain point can be determined easily.

(See color plates, p. CP-5.)

6 Discussion

In this paper a probing tool for the interactive visualization of local flow is presented. It gives a researcher the possibility to investigate details of the flow and is complementary to other methods of flow visualization. Additional tools such as animation and the visibility of multiple probes offer the possibility of global insight in the flow. Mapping of the tensor on geometric objects simplifies understanding of the abstract data. Because current graphics hardware allows fast rendering of polygons, interactive speeds are easily achieved. The same principle could be applied to other types of multi-variate data sets, but possibly requires the design of appropriate sets of primitives.

Some features of the tensor cannot yet be extracted from the probe. An example is dilatation. A valuebox could be used for that, but a solution where those values are represented in the probe would be better. In this paper we assumed time independent flow. In practice many simulations produce time dependent data. Extending the probe for use in such data requires that spatial and temporal derivatives are considered.

At critical points [6], where the magnitude of the velocity vanishes, it is impossible to construct a local frame of reference. In these points another representation for the local flow has to be used. A possibility is the use of special types of icons for each type of critical point (vortex, sink, source, saddle or combination of those).

The probe can be used in combination with other visualization techniques. A global visualization technique could be used to show the overall structure of the data while the probe is used to investigate certain regions of the data in detail.

Acknowledgements

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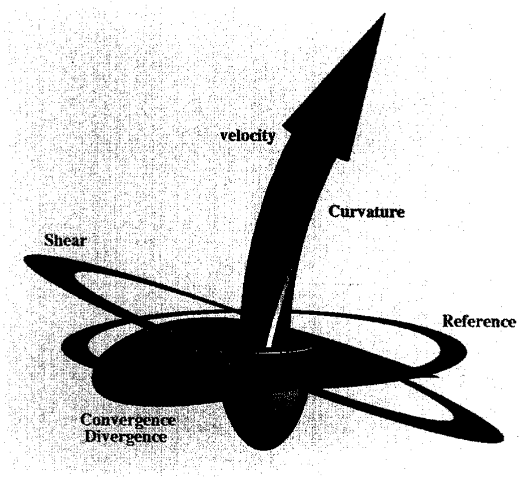
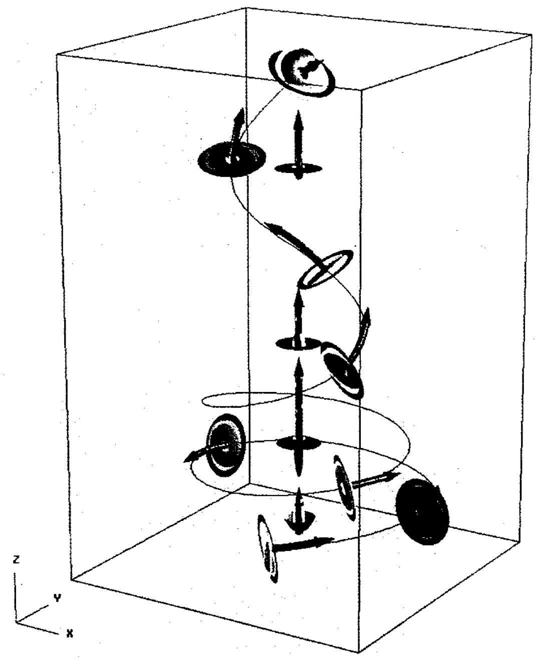


Figure 3: Components of the probe



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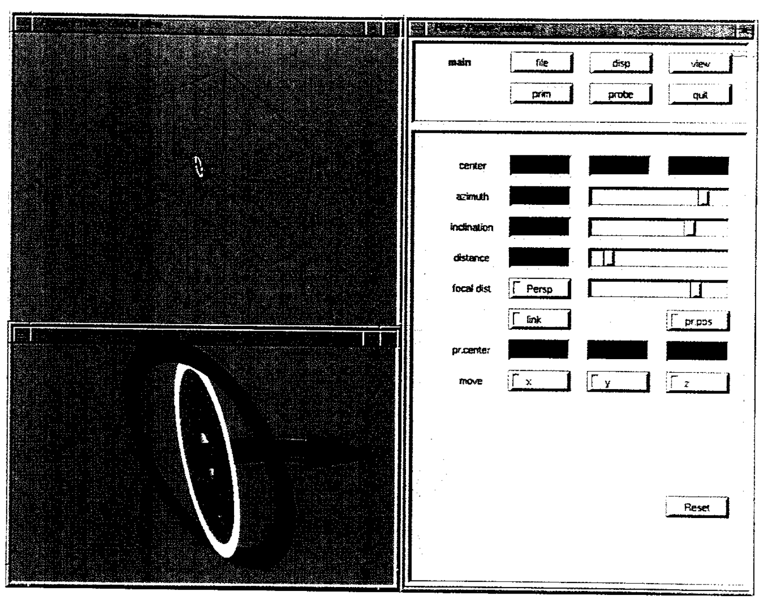

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